
# ATTACHMENT GUIDE: MATHS TUNE UP!

The contents of the combined attachments in this report are as follows:

1. Resource sheets
2. Poster
3. Postcard
4. Media release
5. Evaluation strategy
6. Online evaluation
7. Practice guide
8. Resource slides
9. Lecturer guide

## Before You Watch

This video shows you how to interpret a linear equation and graph it on a Cartesian plane. It will refresh your memory of how to use a Cartesian plane and develop your understanding of what the graph of a linear equation means.

This topic also builds on the fundamental concepts of algebra covered in **Introduction to Algebra**. So, if you're unsure about algebra in general, watch that video first, then come back.

## The Video Content

You will often see equations that look like this:

- $y = 3x + 2$
- $7 = 2x + 2y$
- $117 = 70n - 4r$

Each of them has two letters, and both letters are only to the power of 1. These are called linear equations. These equations are common in the real world, as they represent things like cruising at a constant speed, or business earnings from the number of items sold.

This type of equation can be graphed. Let's consider the first example as a question:

Graph the equation
$y = 3x + 2$

So, what is the question asking?

**Step 1 Understand the question**

Why are we asked to graph? Why not just solve it like we did in Introduction to Algebra? In that video, we solved the equation by getting the unknown (the letter) on one side of the equals sign, and everything else on the other side. But we can’t use the same method in this situation, because there are *two* unknowns.

What happens if we try to solve it? Is:

$x = 1$

a possible solution? If we substitute $x = 1$ into the equation y = $3x + 2$ we get:

$y = 3 \text{ x } 1 + 2$

$= 5$

So yes, it is. Now, can:

$x = 0$

be a solution? If we substitute $x = 0$ into the equation $y = 3x + 2$ then we get:

$y = 3 \text{ x } 0 + 2$

$= 2$

Another solution! Can:

$x = 2$

be a solution to the equation $y = 3x + 2$?

$y = 3 \text{ x } 2 + 2$

$= 8$

Yes it can. As you can see, $x$ can be any number we like, and for each value of $x$, there is a corresponding value $y$ for which ($x$,$y$) is a solution to this equation.

So if $x$ can be anything, when $x = 1$ then $y = 5$, and we have the point (1,5).
When $x = 2$ then $y = 8$, and we have the point (2,8), and so on.

So how can we graph it?

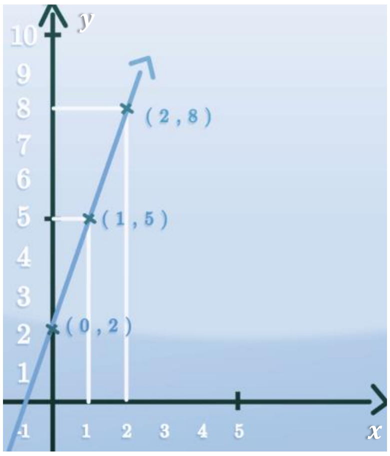
**Step 2 Develop a plan**

Let’s try graphing all the points. This is our plan!

**Step 3 Carry out the plan**

We can plot these 3 points in the Cartesian plane:

$(x = 1, y = 5)$
$(x = 2, y = 8)$
$(x = 0, y = 2)$




All these points lie on a straight line.

In fact, linear equations *always* produce a straight line, hence their name, *linear*. So, to graph a linear equation, we just need to find two points that are a solution to the equation – any two points – and draw a line through them. This line represents *all* the possible solutions to this equation.

**Step 4 Reality Check**

Is our answer reasonable?

If you have only used two points to plot a line, a simple way to check if your graph is correct is to pick a new solution to the equation, and check that it is also on the line. Let's try a new solution for $y = 3x + 2$. So, if:

$x$ = -1

then:

$y$ = 3 x -1 + 2
= -3 + 2
= -1

The point is $(x = -1, y = -1)$. And yes, this point lies on our line, which is a good indication that our answer is correct.

Linear graphs can represent a lot of different situations, so you'll regularly come across them.

# *Some Practice Questions*

Graph:

1. $7 = 3x + 2y$

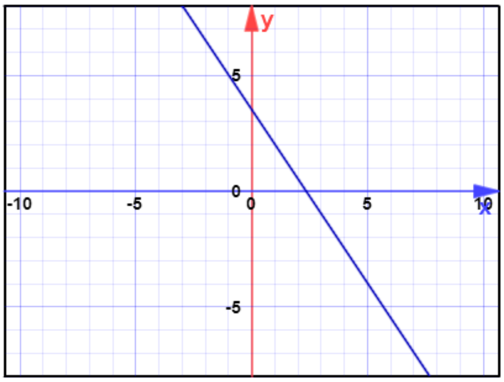
2. $117 = 70n - 4r$ where $n$ is the vertical axis and $r$ is the horizontal axis

3. $0.5p = r + 7$ where $p$ is the vertical axis and $r$ is the horizontal axis

4. $3 = x/y$
(Tip: can you rearrange this equation to look like a linear equation?)

Answers

1.

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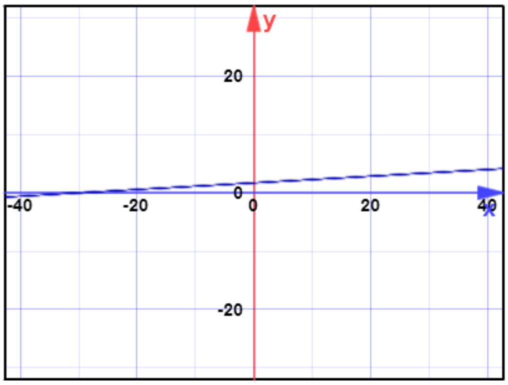

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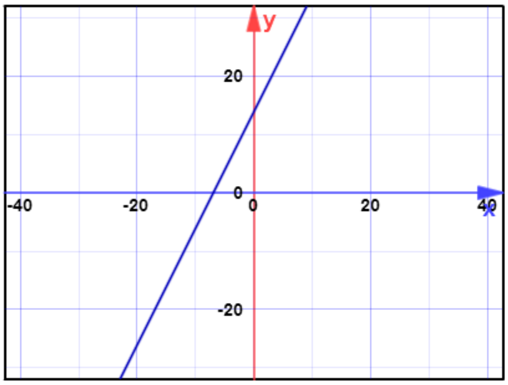


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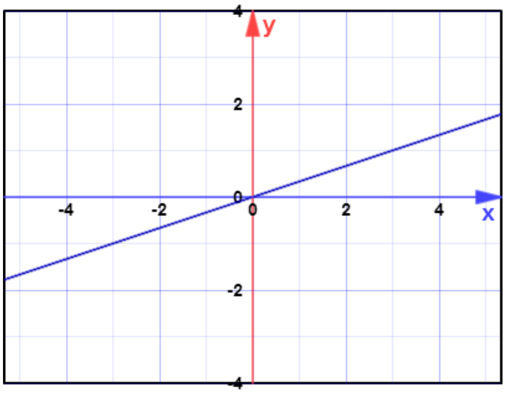


Take a look at the working out for each answer **here**.

## Now What?

Linear equations are widely used to model situations, so they are an important category of graph to understand. Once you're comfortable graphing linear functions, then it's a good idea to look at how you can interpret word problems and create a linear equation from the description. Some lessons on how to do this can be found here:

- https://www.khanacademy.org/math/algebra-basics/core-algebra-linear-equations-inequalities/core-algebra-linear-equation-word-problems/v/linear-equation-word-problem-example

More example questions are available here:

- https://www.ixl.com/math/algebra-1/solve-linear-equations-word-problems

When you're confident with creating, graphing and solving linear equations, the next logical step is to look at situations where two (or more) linear equations are used together to find a solution. This is shown in **Simultaneous Equations**.

## But When Am I Going To Use This?

Linear equations are very commonly used in everyday life to model situations, so you will run into them a lot. Some simple examples of applications of linear functions include:

- costing a phone call or taxi ride
- calculating revenue and expenses for a company against items sold
- doubling or halving a recipe for cooking
- feeding a group of people at a party
- working out driving time against distance to travel.

## Other Links

**Maths is Fun** has a lot of useful applets and summaries for a wide variety of mathematics subjects. One applet that is particularly relevant for this topic is their equation grapher. You can enter a mathematical equation (in terms of $x$ and $y$, so if your letters are different, just change them to $x$ and $y$) and it will graph that equation. Keep in mind, however, you will still need to be able to graph them with pen and paper yourself!

- http://www.mathsisfun.com/data/grapher-equation.html

**Fort Bend Tutoring** has a series of YouTube videos covering numerous different mathematical concepts. This specific video covers graphing a linear equation by finding the $x$ and $y$ intercepts (a method very similar to what we did here).

- https://www.youtube.com/watch?v=zuwrIO5_vL0

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The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. The first link below is to an entire chapter dedicated to linear equations. It includes lots of videos, examples and quizzes. The second link leads to a demonstration of how to interpret a word problem in terms of a linear equation.

- https://www.khanacademy.org/math/algebra/two-var-linear-equations-and-intro-to-functions
- https://www.khanacademy.org/math/algebra-basics/core-algebra-linear-equations-inequalities/core-algebra-linear-equation-word-problems/v/linear-equation-word-problem-example

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. This content shows how to graph a linear equation using two different methods: finding the $x$ and $y$ intercepts (similar to what we did here), and the point and slope method.

- http://patrickjmt.com/graphing-linear-functions-by-finding-xy-intercept/
- http://patrickjmt.com/graphing-a-line-using-a-point-and-slope/

# Factorisation of Algebraic Expressions

## Before You Watch

This topic covers how to factorise certain algebraic expressions and how to use them. It builds upon a basic understanding of:

- algebra
- how to use and expand brackets
- indices in algebra.

These concepts are explained in **Introduction to Algebra** and **Indices Laws**. If you're not confident in the above areas, watch these two videos first, to make sure you have the background knowledge needed, then come back.

## The Video Content

Our first example is to factorise $2x + 3x^2$.

So what does 'factorise' mean?

**Step 1 Understand the question**

Factorising an algebraic expression is the opposite process of expanding the brackets. It works in exactly the same way as finding factors of numbers. When you expand the brackets, you multiply the terms outside the brackets with each of the terms inside the brackets to get one expression (which can have + and / or - symbols in it).

For instance, let’s expand:

$$4y(2y + x)$$

We do this by multiplying both of the terms in the bracket by $4y$:

$$8y^2 + 4xy$$

So to go from:

$$4y(2y + x)$$

to:

$$8y^2 + 4xy$$

is to expand. To go the other way is to factorise. We can now say that $4y$ and $2y + x$ are factors of the expression $8y^2 + 4xy$.

This process is called factorisation because the equation is being expressed as the multiplication of a series of factors. For example, we can factorise the number 15 by expressing it as a multiple of its prime factors: 15 = 3 x 5.

Back to the original question. How can we factorise $2x + 3x^2$ ?

**Step 2 Develop a plan**

The simplest method of factorisation is to find a common factor (amongst each of the terms.) In this case we notice that $x$ goes into both terms. So that's what the plan is: to take $x$ out as a factor.

**Step 3 Carry out the plan**

To take $x$ as a factor, we need to first multiply and divide by $x$:

$$x \,/\, x(2x + 3x^2)$$

which doesn't change the value of the equation. Then we manipulate the expression a little to take $x$ out:

$$= x(1 \,/\, x)(2x + 3x^2)$$

$$= x((2x + 3x^2) \,/\, x)$$

Now we can separate this fraction into two fractions, $2x \,/\, x$ and $3x^2 \,/\, x$:

$$= x(2x \,/\, x + 3x^2 \,/\, x)$$

Looking at each of these fractions, we can cross an $x$ off the top and bottom so that our final answer is:

$$x(2 + 3x)$$

That can't be factorised anymore, so we are done. Is our answer correct?

**Step 4 Reality check**

We can easily check a factorisation by expanding out the brackets. If we get back to the original answer, great! This is because factorisation is the opposite of expansion:

$x(2 + 3x)$
$= x \times 2 + x \times 3x$
$= 2x + 3x^2$

**Another example**

Say we are asked to factorise $4a^3b^2 + 8ab^4 + 2ab$.

In this case, first we notice that the biggest or highest common factor is $2ab$.
So, as before, we take $2ab$ out the front, and divide everything inside by $2ab$.

***Did you know?***

In the above example, how do we know that $2ab$ is the highest common factor? Here are the steps.

*Step 1*

Look at the numbers first. Find the highest common factor (HCF) of the numbers at the front. In this example the numbers are 4, 8 and 2. The highest common factor of these is 2.

*Step 2*

Look at the letters. A letter is only a common factor if it appears in *all* the terms. For each letter that is in all the terms, we put it to the *lowest* power that appears. In this example, both the letters "$a$" and "$b$" appear in all three terms. The letter "$a$" has powers 3, 1 and 1, so we need to put "$a$" to the power of 1. The letter "$b$" has powers of 2, 4 and 1, so we need to put "$b$" to the power of 1.

*Step 3*

Put it all together. We can see that our highest common factor is the number 2, and the letters "$a$" to the power of 1 and "$b$" to the power of 1. So the highest common factor is $2ab$.

So, working with the highest common factor, take $2ab$ out the front, and divide everything inside by $2ab$:

$$2ab((4a^3b^2 + 8ab^4 + 2ab) / 2ab)$$

Now break up that fraction into three fractions:

$$2ab((4a^3b^2 / 2ab + 8ab^4 / 2ab + 2ab / 2ab)$$

Then cross off all the common terms top and bottom for the three different fractions. The final result is:

$$2ab(2a^2b + 4b^3 + 1)$$

We've reached the point where the term inside the bracket can't be factorised any more. The easiest way we can see this is because one of the terms in the bracket is 1. The number 1 has no factors other than itself, so we can't factorise it further.

Can you do the reality check? Do you get to the original expression if you expand this bracket?

Multiply each term inside the brackets by $2ab$:

$$2ab(2a^2b) + 2ab(4b^3) + 2ab(1)$$

Then expand:

$$4a^3b^2 + 8ab^4 + 2ab$$

This matches the original expression so, yes, the answer is correct.

## Some Practice Questions

Factorise the following expressions:

1. $3k^8 + 2k^4$
2. $4a + 4b$
3. $b^2j^7 + j^5m^3$

4. $6k^2 + 12k^3m$

5. $15p^4t^3y^2 + 20k^3p^4z^3$

6. $2a^2b - 2^8ab$

7. $4x^2 + 8xy$

8. $6m - 3mn + 12km$

Answers

1. $k^4(3k^4 + 2)$

2. $4(a + b)$

3. $j^5(b^2j^2 + m^3)$

4. $6k^2(1 + 2km)$

5. $5p^4(3t^3y^2 + 4k^3z^3)$

6. $2ab(a - 14)$

7. $4x(x + 2y)$

8. $3m(2 - n + 4k)$

Take a look at the working out for each answer **here**.

## Now What?

The method presented here is the simplest method of factorisation – identifying a common factor. Two other methods often used are factorisation by grouping, and the difference of two squares. Factorisation is a valuable skill and it's useful to also become familiar with these other methods. Further resources are provided in the Other Links section.

Once you are confident with factorisation using the method covered here, you can examine other algebraic skills, such as **Algebraic Fractions** or **Linear Equations**.

## *Other Links*

If you have difficulty finding the highest common factor in algebraic expressions, this video tutorial at **Virtual Nerd** gives a very thorough method of determining the highest common factor. The site also has other useful tutorial videos.

- http://www.virtualnerd.com/algebra-2/quadratics/solve-equations-by-factoring/factoring-strategies/greatest-common-factor-example

**Everything Maths** has a good summary of different factorisation methods as well as a wide selection of worked examples and questions with answers.

- http://everythingmaths.co.za/maths/grade-10/01-algebraic-expressions/01-algebraic-expressions-06.cnxmlplus

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the relevant section. At the top of the page is the topic “Introduction to Factorization” followed by further topics on factorisation.

- https://www.khanacademy.org/math/algebra/polynomial-factorization

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Patrick JMT uses the term “factoring” and there is a very wide selection of factoring videos available covering numerous methods.

- http://patrickjmt.com/

# Indices Laws

## Before You Watch

This video deals with numbers called indices, specifically as applied to algebra. We often call them powers, or exponents. An example is the number $3^5$, which we read as "three to the power of five". This means three times itself five times:

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3$$

Here the number 5 is called the exponent or the index. The plural of index is indices. This definition of indices, or exponents, is well demonstrated here:

- http://www.mathsisfun.com/algebra/exponent-laws.html

This topic builds on the fundamental concepts of algebra covered in **Introduction to Algebra**. If you need a refresher on algebra in general, watch that video first, then come back.

## The Video Content

This topic covers some of the basic laws of indices, and looks at how they apply to algebra.

To refresh our memories about indices we'll start with the example $4^3$. We refer to the number 3 as an index. We know that this means 4 multiplied by itself 3 times:

$$4^3 = 4 \times 4 \times 4$$

It's the same for algebra:

$$b^3 = b \times b \times b$$

What happens when we multiply these terms together?

Consider:

$$k^3 \text{ x } k^2$$

This means $k$ multiplied by itself 3 times, then multiplied by $k$ another two times. So it's equal to $k$ to the power of five:

$$k^3 \text{ x } k^2 = k^5$$

If we multiply, using the same letter, we can just add the powers.

Another example:

$$a^4 \text{ x } a^3 = a^{(4+3)}$$
$$a^4 \text{ x } a^3 = a^7$$

What about division?

Let's say, for example:

$$f^4 \,/\, f^2$$

That's $f$ multiplied 4 times, divided by $f$ multiplied twice. We can simplify this fraction by crossing off two $f$'s from both the top and bottom. That leaves us with $f$ multiplied twice, in other words $f$ squared:

$$f^4 \,/\, f^2 = f^2$$

As we can see from this, what really happened is that we subtracted the indices!

Another example:

$$a^{13} \,/\, a^5 = a^{(13\text{-}5)}$$
$$a^{13} \,/\, a^5 = a^8$$

What about when the equation has more than one letter in it? Say:

$$n^3m^5 \text{ x } n^4m^2 \,/\, n^2m^3$$

As is the case for only one letter, we add the indices when multiplying and subtract the indices when dividing – but this time it's done separately for each letter. So, the above example is equal to:

$$n^{(3+4\text{-}2)}m^{(5+2\text{-}3)} = n^5m^4$$

Finally, let's look at expressions like:

$$(r^3)^2$$

Just as with numbers, we do the part inside the brackets first:

$$(r^3)^2 = (r \times r \times r)^2$$

and then multiply as many times as the index indicates: in this case, twice.

So this is equal to:

$$(r \times r \times r)^2 = (r \times r \times r) \times (r \times r \times r)$$

which is $r$ multiplied 6 times:

$$(r \times r \times r) \times (r \times r \times r) = r \times r \times r \times r \times r \times r$$

which is $r$ to the power of 6:

$$r \times r \times r \times r \times r \times r = r^6$$

In other words, the powers are multiplied. So:

$$(y^5)^{11}$$

is simply:

$$(y^5)^{11} = y^{(5 \times 11)} = y^{55}$$

## *Some Practice Questions*

1. $3k^8 \times 2k^4 =$

2. $10y^{11} / 5y^3 =$

3. $b^2j^7 \times j^5m^3 =$

4. $r^8 \times r^3 / r^2 =$

5. $(p^4)^3 \times p^9 =$

6. $(w^7)^2 / (w^3)^3 =$

7. $q^6t^5f^3 \times q^8f^2 / t^2f =$

8. $a^7h^2 / (a^2)^3 \times a^5h^4 =$

Answers

1. $6k^{12}$
2. $2y^{8}$
3. $b^{2}j^{12}m^{3}$
4. $r^{9}$
5. $p^{21}$
6. $w^{5}$
7. $f^{4}q^{14}t^{3}$
8. $a^{6}h^{6}$

Take a look at the working out for each answer **here**.

## Now What?

Comfortable with the concepts covered in this topic? A great next step would be to look at **Negative and Fractional Indices** for a more complete picture of the rules surrounding indices.

## But When Am I Going To Use This?

Indices are used in many different situations in real life. A common example is in writing very large or very small numbers. These are often written in scientific notation, and can be stored in computers as a type of variable known as a floating point variable. Scientific notation makes heavy use of indices to keep numbers easier to work with. Floating point variables are very important in all areas of computing, including gaming physics.

Indices are also used in the calculation of areas and volumes. For example, the area of a square is the length squared, and the volume of a cube is the length cubed. This is especially important when changing units of measurement, such as from cubic metres to cubic centimetres.

Plus, indices are used in certain kinds of other measurements, including acidity (pH), the loudness of sound (decibels), or the intensity of earthquakes (the Richter scale). All of these measurements use what is known as a logarithmic scale, which relies on indices.

## Other Links

**Maths is Fun** has a useful applet to help you understand the basic idea of indices, followed by an easy to follow summary of the rules.

- http://www.mathsisfun.com/algebra/exponent-laws.html

Another page with a lot of interactive maths problems is **www.intmath.com**. It is well structured at the top level and features are easy to find.

- http://www.intmath.com/exponents-radicals/exponent-radical.php

**Laerd Mathematics** gives a succinct summary of the rules, and follows this up with a wide selection of questions. Worked answers are available.

- http://mathematics.laerd.com/maths/indices-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the relevant section for this topic.

- https://www.khanacademy.org/math/algebra-basics/core-algebra-exponent-expressions/core-algebra-exponent-properties/v/exponent-properties-involving-products

**Patrick JMT** (Just Maths Tutorials) has many video tutorials covering a large range of mathematical concepts. Here are two useful videos: the first one covers the basic indices rules, and the second provides example questions.

- http://patrickjmt.com/basic-exponent-properties/
- http://patrickjmt.com/exponents-applying-the-rules-of-exponents-basic-ex-1/

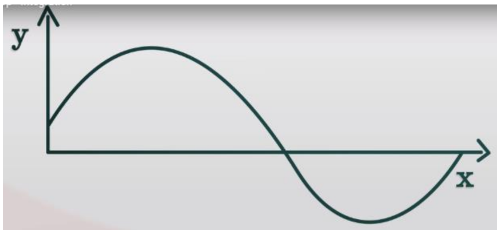
# Integration

## Before You Watch

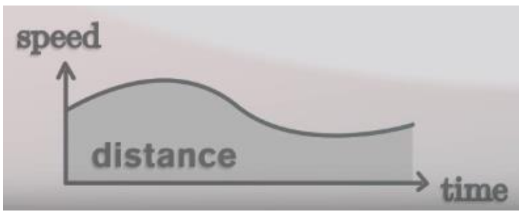
Before watching this video, make sure you've seen **Introduction to Calculus**. In fact, even if you've seen it already, it's a great idea to watch it again! A lot of the notation introduced in that video is used in this one. This topic also builds on the key concept of calculus that was explained in the introduction. So we suggest you watch **Introduction to Calculus**, then come back.

## The Video Content

As we saw in Introduction to Calculus, calculus is about the concept of infinitesimal change in one quantity compared to another quantity, for example, temperature with time. Let's look at how we can use this concept to help us find the area under a curve. Consider this curve. The vertical axis is $y$ and the horizontal axis is $x$.



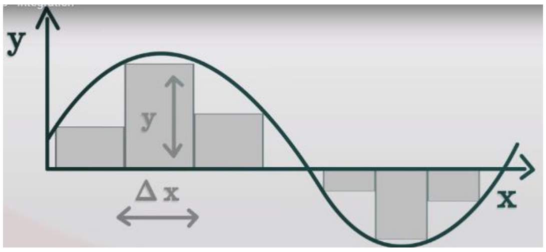

An example of where this is used could be if the $y$ axis is speed and the $x$ axis is time. Then the area under the curve is distance travelled. This helps us determine how far a car has travelled, only knowing its speed over time.



To find the area, we could start by making a series of rectangles under the curve, then add the area of all the rectangles. That would give an approximation of the total area.

From the last video we saw that we often use the capital Greek letter Δ to represent change, so we can express the width of these rectangles as the change in $x$, $\Delta x$.
The height of each rectangle is the value of $y$ at that point.
So:

area of each rectangle = $y$ x $\Delta x$

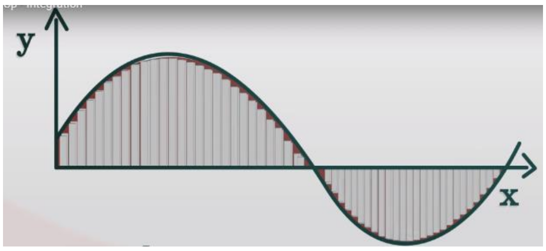



The area of all rectangles together is, therefore, the sum of these rectangles.
The Greek letter Σ is usually used to mean 'sum'.
So:

total area = Σ $y$ x $\Delta x$

But we can see that we are missing out on a lot of area. How can we cover more of the area?

If we were to make $\Delta x$ smaller, the rectangles get narrower. That way, the area gets more accurate and we cover more of the area under the curve.




To get the exact area, we make the rectangles smaller and smaller and, in doing so, you can see that the area gets closer and closer to the real area under the curve:

Σ $y$ x $\Delta x$

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In fact, if we make the rectangles infinitely narrow, the area of the rectangles *is* the area under the curve. Just like in Introduction to Calculus, the $\Delta x$ is getting infinitesimally small.

To represent an infinitely small step in $x$, we use $dx.$

However, as the rectangles become infinitely thin, there also becomes infinitely many of them. So we replace the $\Sigma$ with a new symbol, the integral symbol ∫, to represent that it is an infinite sum.

This is called the integral of $y$ times $dx$:

$$\int y * dx$$

It means the infinite sum of rectangles of height $y$ and width $dx$.

This is an example of the core concept of integral calculus.

Looking back, we can see that we broke down our complex shape – the curve – into simple shapes we knew the area of. Then we imagined making those simple shapes infinitely small (or narrow, in this case) so that the area of all the simple shapes is the same as the complex shape.

As you will find out, this process has many potential applications in engineering and science.

## Now What?

This video builds on **Introduction to Calculus** and discusses the core ideas of integral calculus, which is one of the two main branches of calculus. If you haven't already done so, work through the videos covering the other branch of calculus, differential calculus. Start with **Rates of Change and Differentiation**.

Alternatively, if you've already seen those videos, the next step is to build up your ability to differentiate. You can do this with the Khan Academy at https://www.khanacademy.org/math/differential-calculus/taking-derivatives.

It is easiest to learn how to integrate by first learning to differentiate. This is because integration is the inverse operation to differentiation, in the similar way that division is the inverse operation to multiplication.

## But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. It has enormous applications in all areas of engineering and science, and is necessary knowledge to study for a degree in engineering or science.

Specifically looking at integral calculus, integration allows us to move from a rate of change and convert that into an absolute quantity. For example, calculus allows the measurement of a car's speed over time (using it's speedometer) to then be converted into distance (as measured by the odometer). Another example is the measurement of the speed of water flowing through a pipe, which can then be used to calculate the total amount of water that flowed through the pipe.

## Other Links

**Maths is Fun** has a great page that covers the basic concepts of integration. It then extends your learning by demonstrating how to perform the integrations, which is an important skill.

- https://www.mathsisfun.com/calculus/integration-introduction.html

**IntMath** gives a good explanation of the process of integration, as well as covering the methods of integration. It provides some excellent examples of applications of calculus that are in common use today, and includes applets to help your understanding of both differential and integral calculus.

- http://www.intmath.com/integration/integration-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical and other concepts, as well as questions to test your knowledge. This content provides a whole chapter on taking the derivatives, including of harder equations not covered in this video.

- https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals

**Patrick JMT** (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This content covers the basic concept of integration. The site also offers a wide selection of other videos covering several integration topics and techniques.

- http://patrickjmt.com/the-definite-integral-understanding-the-definition/

# Introduction to Algebra

## Before You Watch

This video introduces algebra. It explains what algebra is, and reinforces how we can manipulate and rearrange algebraic equations around the equals sign. It's a good idea to watch this video before viewing the other algebra videos.

This video uses the 4-step problem-solving method we covered in **Introduction to Problem-Solving**. So, if you haven't watched that video yet, start there, then come back.

## The Video Content

Algebra is all about using letters to represent values that we don’t know. Sometimes those letters can be found in equations, and we need to rearrange the equation to work out what number the letter represents.

Consider this equation:

$$x + 15 = 2x + 3$$

First, let’s look at what the equals sign in the middle means.

**Step 1 Understand the question**

An equals sign means that what is on *one side* of the symbol is equal to what is on *the other side* of the symbol.

An equation is like a balanced set of scales. Considering $x + 15 = 2x + 3$, we could say that the scales are balanced if we have one box and 15 kg of weights on one side, and two boxes and 3 kg of weights on the other side. We don’t know what the boxes weigh – we have called it $x$ here – but it could be anything. We could just put an empty square there to fill in later, or even a smiley face.

We can do anything we want with the scales, *as long as we do the same to both sides*. We could add or subtract weight, or add boxes – anything – *as long as we do the same to both sides*.

**Step 2 Develop a plan**

To solve an equation – that is, to work out what the unknown quantity is – we need to have the unknown by itself on one side, and a number on the other.

Mathematically, like this:

$x$ = *a number*

**Step 3 Carry out the plan**

What can we do that will get our equation closer to what we want, which is $x$ on one side and a number on the other?

Let's start by removing the $x$ from the left side. If we subtract $x$ from the left side, we have to do the same to the right side or the scales won't be balanced:

$$x + 15 - x = 2x + 3 - x$$

The $x$'s on the left cancel each other out, and $2x - x$ is just $1x$, so we have:

$$15 = x + 3$$

Now we subtract 3 from both sides:

$$15 - 3 = x + 3 - 3$$

$$12 = x$$

Therefore the answer is:

$$x = 12$$

**Step 4 Reality check**

Is our answer of 12 correct?

We can check by substituting $x$ = 12 into the original equation to see if the sides match. The original equation was:

$$x + 15 = 2x + 3$$

After the substitution, this becomes:

$$12 + 15 = 2 \times 12 + 3$$

Adding up the numbers:

$$27 = 24 + 3$$
$$27 = 27$$

The two sides balance. Yes, 12 is the answer!

### Did you know?

The letter $x$ is often used in algebra, far more than any other letter (even though we can use any letter we like). Why do we use $x$ so much? The popularity of $x$ in algebra appears to have originated with the French Mathematician and Philosopher René Descartes, who, in his landmark work *La Géométrie*, used the letters at the start of the alphabet – $a$, $b$ and $c$ – to represent known values, and the letters at the end – $x$, $y$ and $z$ – to represent unknown values. Why did he do this? Nobody really knows. One suggestion is because $x$, $y$ and $z$ aren't used much in French, and so there were lots of spare $x$'s for the printing press. Another suggestion is based on a poor translation of the arabic word "al-shalan", which means "the unknown thing". So, in essence, the reason we now use $x$ so widely in algebra is because a Frenchman did so in his book a long time ago.

## Some Practice Questions

Find the value of the letter in the following equations:

1. $x + 3 = 6$

2. $3k = 12$

3. $9p$ - 14 = $2p$

4. $3r$ + 3 = 23 - $2r$

5. $2t$ = 5

6. $6w$ + 34 = $3w$ - 11

7. $n$ - 59 = $3n$ +27

8. $38q$ + 564 = $43q$ - 392

Answers

1. $x$ = 3

2. $k$ = 4

3. $p$ = 2

4. $r$ = 4

5. $t$ = 2.5

6. $w$ = -15

7. $n$ = -43

8. $q$ = 191.2

Take a look at the working out for each answer **here**.

## Now What?

This video is the launching pad for the entire subject of algebra. Now you can move on, further developing your skills and learning different ways of manipulating algebraic equations. For instance, you can discover how **indices** or **fractions** work with algebra, or look at one of the most popular algebraic equation types, the **linear equation**.

## But When Am I Going To Use This?

Algebra is one of the foundation areas of mathematics. It is essential for any study in science, mathematics, engineering and many other fields. You will find yourself using it consistently at university and beyond.

## Other Links

**Maths Is Fun** has some great content that illustrates the set of scales analogy for the equals sign. It also covers how to add and subtract, and how to multiply and divide, from both sides of the equals sign.

- http://www.mathsisfun.com/algebra/add-subtract-balance.html
- http://www.mathsisfun.com/algebra/introduction.html
- http://www.mathsisfun.com/algebra/introduction-multiply.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to a chapter that covers introduction to algebra, as well as explaining the importance of doing the same thing to both sides of the equals sign.

- https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/why-of-algebra/v/why-we-do-the-same-thing-to-both-sides-simple-equations

# Introduction to Calculus

## Before You Watch

This topic introduces calculus. It presents the fundamental premise of calculus as well as some of the unique notation that is used in calculus in preparation for the other videos in this series.

You don't have to know anything about calculus before watching this video, however, it is a good idea to be familiar with the general concepts of algebra first. If you're not confident with rearranging **algebraic equations** or working with **algebraic fractions**, review the videos on those topics, then come back. You also need to be familiar with the **linear equation** before proceeding with calculus.

## The Video Content

This video talks about the core concept of calculus, and explains some of the terminology. Calculus is useful when there are two quantities varying in relation to each other. The example we'll discuss here is distance varying with time.

Let's think about a moving car.

Remember from school that:

speed = distance / time

To calculate the car's speed, we'd measure the distance it travelled, and divide that by the time that passed while travelling. For instance, if it travelled 60 km in an hour, the average speed was 60 km/hr.

Another way of thinking about *distance* is the *change* in the car's *location*.

In mathematics, we often use the Greek capital letter delta, or $\Delta$, to represent change.

So, in this example, if we call position $x$ and time $t$, then:

speed = change in location / change in time

or:

$\Delta x$ / $\Delta t$

This is just another way of expressing the speed, time and distance formula you would have used in school.

But we usually don't want to measure speed over the last hour or minute; we want to measure speed now, *at this instant*.

The traditional 'speed = distance / time' formula only gives us the average speed over some period of time. It does not calculate the speed now*, at this very moment.*

This is where we turn to calculus.

Calculus tells us that we need to imagine calculating the speed over smaller and smaller steps in time, over, say, the last half hour, then the last minute, the last second, the last hundredth of a second, and so on.

Our speed is still 'change in position divided by change in time', but as we calculate it over smaller and smaller steps, both the change in position and the change in time get smaller.

Then, as we look at smaller and smaller steps in time, the speed being calculated is getting closer and closer to our speed this instant.

Now imagine continuing until the change in time and the change in position become infinitesimally small. This speed, this *infinitely small change in position divided by the infinitely small change in time,* is the speed this *instant.*

To represent the changes which are now infinitely small, we replace the Greek capital letter delta $\Delta$ with a lower case $d$, so:

instantaneous speed = $dx$ / $dt$

Calculus is all about this process of imagining the changes in distance and time getting smaller and smaller until they become infinitesimally small.

*In Calculus,* $d$ *means an infinitesimally small change.*

In our example $dt$ is a tiny step in *time* and $dx$ is a tiny step in *position*, but it could be anything.

There are two main types of calculus: differential calculus and integral calculus. What we've just discussed was mainly based on differential calculus. The other topics in this series look at each of the two kinds separately.

## Now What?

Now that you've been introduced to the fundamental concepts of calculus, you can explore further. As mentioned, there are two main categories of calculus: differential calculus and integral calculus. Most commonly, differential calculus is taught first, followed by integral calculus.

So, when you're familiar with the basic concepts in this video, watch **Rates of Change and Differentiation** next. Even if you already know how to differentiate, it will help you understand what differentiation means.

## But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. For instance, velocity (or speed) is a change of position over a change in time, and acceleration is a change in velocity over a change in time – so any motion is studied using calculus. Other examples include the flow of water through pipes over time, or changing commodity prices against demand. Because change is everywhere, the potential applications for calculus are endless, particularly in engineering and science. Calculus is necessary knowledge for any degree related to engineering or science.

# Other Links

**Maths is Fun** has a great page that takes you through a simple problem which highlights the need for calculus to discuss changes happening around us. It then continues to explore the main two areas of calculus, differentiation and integration, and provides regular questions to test your understanding.

- https://www.mathsisfun.com/calculus/introduction.html

**IntMath** gives a bit of historical perspective to explain the sometimes confusing notation that is used in calculus, discussing how it is the mixed product of two mathematicians working independently. It also provides some excellent examples of applications of calculus that are in common use today, as well as helpful applets to understand both differential and integral calculus.

- http://www.intmath.com/calculus/calculus-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical topics, as well as questions to test your knowledge. This content explains the historical development of calculus, and is also an excellent introduction to differential calculus and the concepts it is based around. From here you can further investigate differential calculus.

- https://www.khanacademy.org/math/differential-calculus/taking-derivatives/intro_differential_calc/v/newton-leibniz-and-usain-bolt

**Patrick JMT** (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This video introduces and explains the concept of a limit to help develop your understanding of this idea.

- https://patrickjmt.com/what-is-a-limit-basic-idea-of-limits/

# Introduction to Probability

## Before You Watch

This topic provides a brief introduction to probability and one of the common methods used to solve probability problems. You don't need any previous knowledge in probability, but you do need to be familiar with the 4-step problem-solving method. So if you haven't already watched **Introduction to Problem-Solving**, do that first. Also, because it will help to be comfortable with some of the fundamental ideas from algebra, it's a good idea to watch **Introduction to Algebra** to refresh your memory, then come back.

## The Video Content

This video introduces the concept of probability and how to calculate the probability of a particular outcome.

Here's a simple example: let's calculate the probability of flipping 2 heads in 2 tosses of a coin.

**Step 1 Understand the question**

What do we mean by the probability of 2 heads?

Let's say that we have a certain number of possible outcomes, or results. For example, rolling a die has 6 possible outcomes, 1 through to 6. And let's say they are all equally likely. This is because when rolling a die, each possible result is just as likely as any other.

Usually we are only interested in some of the possible outcomes: for instance, to roll a 6 or, in our example question, to flip 2 heads.

Then the probability ($p$) of the outcome we want is equal to:

$$p = \frac{\text{number of desirable outcomes}}{\text{total number of possible outcomes}}$$

We need to determine:

- how many ways there are of flipping 2 heads and
- how many possible outcomes in total there are.

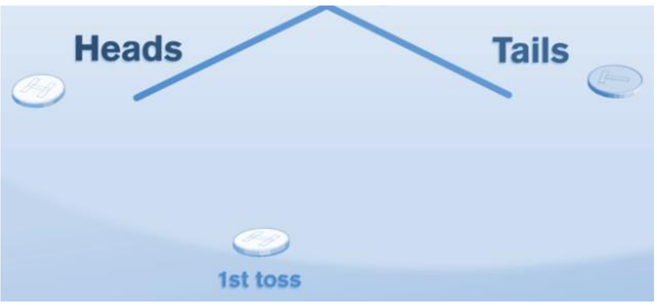
**Step 2 Develop a plan**

In probability, it is often helpful to visualise the situation. Here we will use a visualisation technique known as a probability tree to do this. That's the plan.

**Step 3 Carry out the plan**

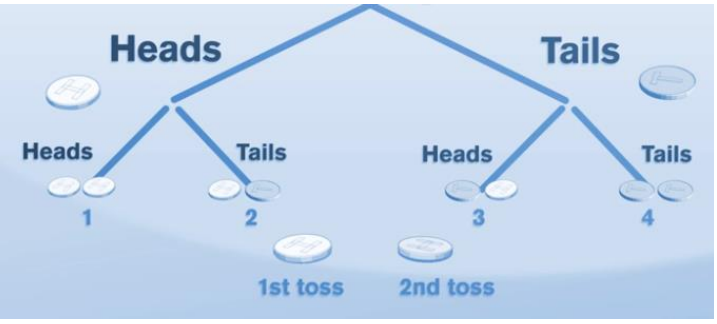
A probability tree starts at a single point, and for each event, it branches out into a number of equally likely possible outcomes. In this case, we can consider the coins being flipped separately, because flipping one coin has no effect on the other coin.

When we flip the first coin, there are 2 equally likely options: heads and tails. Therefore our tree goes into 2 branches, 1 for heads and 1 for tails.




(It is possible to create a probability tree where the branches are not of equal likelihood; however, in this example we have chosen a tree where all branches are equally likely.)

Now we flip the second coin. It also has 2 equally likely outcomes, so each of our 2 branches now branches into 2 more. We have 4 total branches.




Now we can see that there are a total of 4 possible outcomes. Each outcome is equally likely, and only one of the branches has two heads.

Therefore, our probability is equal to the number of desirable outcomes, which is 1, over the total number of outcomes, which is 4:

$p$ = 1 / 4

So our answer is one quarter, or 25%.

**Step 4 Reality check**

It can be difficult to know if a probability answer is correct or not, but we can catch obvious mistakes.

If the answer is bigger than one, or 100%, or if the sum of different probabilities is bigger than one, or 100%, then something went wrong.

For example, the probability of 2 heads plus the probability of 2 tails must be less than 100%.

Since we know that the chance of getting 2 tails has to be the same as 2 heads, then the probability of 2 heads must be less than 50%. We know this because there are also other outcomes for the 2 coin tosses to consider, namely a head and a tail.

Similarly, if the answer is negative, we know that it's wrong. Taking all those things into consideration, 25% seems a reasonable answer.

If we wanted to expand our reality check, we could consider other things:

- If an event has no chance of occurring it has a probability of 0.
- If an event will definitely occur then it has a probability 1 (or 100%).

This tells us we must check that the answer is in the range of 0 to 1 (0 to 100%) – if it isn't, we've made an error.

We also know that including additional criteria will reduce (or at least cannot increase) the probability.

For instance, if the probability of 1 head appearing on a single coin toss is 0.5, and we add the criterion of considering a second coin toss and also getting a head, then the probability of *2 heads* will be *less than or equal to 0.5* – it cannot be greater than 0.5.

## A Practice Question

What is the probability of rolling a 7 when rolling 2 standard 6-sided dice?

Answer

6 / 36 = 1 / 6 = 16.67%

Take a look at the working out for the answer **here**.

## Now What?

Now that you're familiar with the fundamental concepts of probability, take a look at **Introduction to Statistics**. You will see that statistics builds on some of the concepts of probability. Alternatively, you could continue to develop your skills within probability by looking at some of the Other Links and exploring harder problems in probability.

## But When Am I Going To Use This?

Probability is widely used in situations where there are a number of unpredictable events. The most obvious example is that of gambling; casinos and poker machine operators study the probability of people winning and then adjust the returns for those winnings such that they have an extremely high probability of making money. Probability is also used in insurance to determine the premiums that must be set in order for the insurance company to make money.

Another application of probability is in quantum physics. At the very small scale, movements of particles becomes unpredictable; we therefore must use probability to determine the likelihood of a particular event happening, such as, for instance, the probability that an atom will split and trigger the atomic bomb.

## Other Links

**Maths is Fun** provides a summary of the fundamental concepts in probability and explains the meanings of common terms. It also offers a selection of questions to practise on, and explores more complex probability problems.

- https://www.mathsisfun.com/data/probability.html

**Online Math Learning** features a series of lessons and videos covering a wide range of concepts in probability. The link below gives you access to a number of sections. The best place to start is "Samples in Probability".

- http://www.onlinemathlearning.com/math-probability.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to a large chapter dedicated to calculating probability.

- https://www.khanacademy.org/math/precalculus/prob_comb

**Patrick JMT** (Just Maths Tutorials) has many video tutorials covering lots of mathematical concepts. This content investigates simple probability. The site also has numerous videos covering more complex probability concepts.

- http://patrickjmt.com/calculating-the-probability-of-simple-events/

# Introduction to Problem-Solving

## Before You Watch

This video is designed to be the first one you watch. It introduces a 4-step method of solving problems that is used throughout this series of videos. So, if you're new to Maths Tune Up, congratulations, you're in the right place!

## The Video Content

Here is a simple, systematic 4-step approach to solving mathematical questions, often referred to as "problems" in mathematics. This approach can be applied to many different types of questions.

**The 4-Step Problem-Solving Method**

**Step 1** **Understand the question**
Understand precisely what the question is requiring us to do, and identify the key pieces of information provided in the question.

**Step 2** **Develop a plan**
Create a plan to solve the question.

**Step 3** **Carry out the plan**
Follow the plan until you have reached a solution.

**Step 4** **Reality check**
Make sure your answer makes sense within the context of the question.

**Example: the problem**

How many squares of any size are there on a chessboard?

**Step 1 Understand the question**

Let's think about a chessboard. What does the question mean? You may consider the many small squares – 1 square high x 1 square wide – that comprise the checkerboard pattern. Obviously they are squares, but the question says "of any size". So, are there more squares on a chessboard than the obvious small ones that we should consider?

The answer is yes. There are 2 x 2 squares, and there are 3 x 3 squares. We need to also consider that these squares will overlap. So what the question is really asking is: 'how many squares of all different sizes there are in total?'

**Step 2 Develop a plan**

To create a plan we need to ask ourselves several questions.

The first question is 'What information has the question provided?'

We also need to ask ourselves 'What does the question require us to determine, or provide?' The answer to this question is obvious: what is the total number of squares.

But how do we get from one question to the other? That is the purpose of the plan: figuring out how to get from what has *been provided* to what we *need to provide*.

We could use a systematic counting system, but is this the best way? Maybe we could look at how many squares there are across, and how many there are down, and use this to simplify the question.

There are 8 squares across, and 8 squares down. This gives us 64 of the smallest, 1 x 1 squares. We could use this as our method and expand it to include the larger squares as a way of solving the question.

Now that we have a plan we can continue to Step 3.

**Step 3 Carry out the plan**

For the 1 x 1 squares, there are 8 across and 8 down:

8 x 8 = 64

What about the 2 x 2 squares? There are 7 across the top, and 7 down. That’s 49 of the 2 x 2 squares:

7 x 7 = 49

For the 3 x 3 squares, we can fit 6 across and 6 down:

6 x 6 = 36

So, as our square size increases, the number we can fit across and down decreases. Now we can work out the larger squares, including the whole chessboard as one square:

5 x 5 = 25
4 x 4 = 16
3 x 3 = 9
2 x 2 = 4
1 x 1 = 1

Add it all together:

64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 204

and we get 204 squares in total.

We have an answer! But we need to ask ourselves 'is the answer reasonable?' That's Step 4.

**Step 4 Reality check**

We can ask ourselves 'what is a reasonable number of squares?' and also 'what answers are completely unreasonable?'

For instance, if we get a negative number as our answer, like -76, is that reasonable? What about 21 and 3/4? Depending on the question, there are other ways we may catch a mistake, such as if we found there to be more of the larger squares than the smaller ones, or if we noticed an obvious square that our plan is missing.

All of these are ways in which we can hopefully catch mistakes, and go back and find out what went wrong before submitting an incorrect answer.

That's the 4-step problem solving method. It should come in handy for any mathematical questions you encounter.

## Now What?

Think for a moment about what we've just covered. Did you understand the concepts? Do you want to go through it again? A good next step is to check that your foundation in **algebra** is solid. Many areas of mathematics, especially those you need at university, will build upon this basic knowledge

## But When Am I Going To Use This?

The 4-step problem-solving technique is a systematic method of approaching complex questions. In mathematics, such questions are referred to as problems. You'll find this technique useful in and beyond mathematics, wherever problems must be solved.

# Introduction to Proofs

## Before You Watch

This video explains the concept of a mathematical proof and how it is different to a "proof" in other fields. As you will see, the concept of a proof within mathematics is far more rigorous and precise than in other areas or what you probably imagine 'proof' to be in your mind. This video doesn't build on any of the previous topics, so feel free to watch this one straight away.

## The Video Content

In mathematics, we often need to prove precisely formulated mathematical statements.

**What is a proof?**

Mathematical proofs are logical arguments that show the validity of statements.
Proofs must be:

- rigorous
- unambiguous.

In fact, mathematics is distinguished by such proofs, as they are typically not available in other fields.

For instance, in the court of law, one is only expected to prove things "beyond reasonable doubt".

In the sciences, theories can only be supported by evidence and cannot, in general, be proved absolutely. For example, in medicine, clinical trials can only show that new drugs are likely to be effective with high probability. However, there is always a small chance that the outcomes of the trial were misleading and that the drug does not work as it should.

Mathematical proofs, in contrast, are absolute and are not subject to statistical uncertainty.

Since mathematical proofs are rigorous and unambiguous, it is also important that the statement to be proved is precise and that all terms used are formally defined.

**What is not a proof?**

Showing that the statement holds for one particular example is not a proof, as proof needs to hold for all instances of a problem.

For example, the statement:

> "The product of any two consecutive integers is even."

is not proved by:

> "Consider integers 5 and 6; 5 x 6 = 30, and 30 is even."

So how could we go about proving this statement?

We could try to use a "proof by exhaustion" where we would need to check every pair of consecutive integers. Unfortunately, the number of such pairs is infinite, and we would never finish checking!

Therefore we have to think of another strategy.

In general, constructing a proof requires a lot of:

- perseverance
- creativity.

A dead end is not a signal to give up, but rather a sign that we need to attack the problem differently.

Fortunately, there are some common strategies for proving mathematical statements that we can try on our problem.

Common strategies include:

- direct proof
- proof by contradiction
- proof by construction
- proof by mathematical induction

However, given that this is only a brief introduction to proofs, further discussion of these strategies will be the focus of future topics.

## Now What?

Now that you're familiar with the basic concept of a mathematical proof, the next step is to continue to develop your knowledge through the Other Links below.

Alternatively, take a look at some of the other videos available, such as **Introduction to Algebra** or **Introduction to Calculus**.

## But When Am I Going To Use This?

Mathematical proofs are very important in a number of areas, particularly in computer science and simulations. Proofs are also used in research where they can be key to explaining, for example, the difference between an old way and a new way of doing something.

Mathematical proofs, when written down, are a way to keep a record of your understanding and to convey that understanding to others.

## Other Links

**Maths is Fun** introduces a method of proving things called mathematical induction. A simplified explanation is provided along with some clear, basic examples.

- https://www.mathsisfun.com/algebra/mathematical-induction.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to a list of resources which discuss the application of proofs across a number of different areas. Randomly choose a few that sound interesting to you and start exploring the world of proofs.

- https://www.khanacademy.org/search?search_again=1&page_search_query=mathematical+proofs

**Patrick JMT** (Just Maths Tutorials) has many video tutorials covering lots of mathematical concepts. This content investigates proof by induction. Start by choosing Example 1 at the bottom of the screen that the link takes you to.

- http://patrickjmt.com/?s=induction

# Introduction to Statistics

## Before You Watch

In this topic you'll be introduced to some of the basic concepts of statistics, in particular random variables, variation and sampling. It doesn't build on ideas discussed in any of the previous videos, so feel free to watch this one straight away.

## The Video Content

This video discusses the concepts of random variables, variation and sampling.

They say a statistician is someone who can have their head in the oven and their feet in ice and on average they are comfortable. However, the key to statistics is not simply to consider averages but to understand variation. We better understand variation by sampling.

Variation is all about us:

- Consider the bottles of 600mL soft drink you buy. The variation may be small but some will be a little fuller than others.
- Consider the lifetimes of a particular brand of lithium battery, or a light globe, or the composition and taste of two pizzas of the same type, bought from the same location. Or even the pull-off force of connector rods in an engine… in each scenario the items are constructed the same way from the same set of ingredients; however, the end result varies.

The amount of variation is important to quantify and minimise.

Whether it is volume in a bottle, battery lifetimes or connector rod strength, the amount or size of each of these measures will vary depending upon which bottle, battery or connector rod we happen to observe.

Consider testing 3 batteries to exhaustion to measure their lifetimes. With a precise enough measuring instrument, we will always identify varying times to battery failure.

Since the lifetimes (measured in hours, for example) exhibit variability, we consider 'battery lifetime' (which is our measure of interest) to be a *random variable*.

Using notation, the random variable, denoted by $X$, may be considered as assuming values according to the following model:

$$X = \mu + E$$

where $\mu$ is a constant and $E$ is a random disturbance.

So, what does this mean?

The constant, as the name suggests, remains the same with every measurement. However, small changes in the manufacturing process, the environment, test equipment and so on result in varying values for $E$ and hence varying values for $X$, which is the real capacity of the battery.

As we consider more and more batteries we build up a picture of how the variable is distributed. We see how there is variability in $X$. We often need to describe, quantify and ultimately reduce variability.

However, we know that we cannot test all batteries to exhaustion or there would be none to sell. We wish to have an understanding of the variable (lifetime) including the size of variation without testing all items. This is where sampling comes into use.

Sampling is the process of taking a subset of items from a larger group (referred to as a population) in order to infer characteristics about that population.

There are many aspects of sampling that need to be considered... but as this is just a brief introduction to statistics, more about sampling is a topic for another day!

## *Now What?*

Now you should be familiar with the fundamental ideas involved in statistics. You can continue to develop your knowledge of statistics through the Other Links below. Alternatively, take a look at some of the other videos available, such as **Introduction to Algebra** or **Introduction to Calculus**.

## *But When Am I Going To Use This?*

Statistics is essential in the study of systems of situations where there is an unpredictable random element. This includes a huge number of situations, such as any system that involves living things, which always have a degree of unpredictability. In fact, any studies that involve people involve statistics: for example, medical processes, education and economics. Other areas statistics can be applied to include quality control, stars, nature, or how wear and tear affects machinery.

## *Other Links*

**Maths is Fun** provides an index of a range of topics covering probability and statistics. Select the 'Probability and Statistics Index' at the top of the page, then choose the area you wish to explore.

- http://www.mathsisfun.com/data/index.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This content provides a good explanation of how probability and statistics are linked, and allows you to investigate more complex statistical concepts.

- https://www.khanacademy.org/math/probability

# Algebraic Fractions

## Before You Watch

This topic covers how to add fractions involving algebra, when the letters are on the top (the numerator) and when they are on the bottom (the denominator).

Before watching this video, it is recommended that you refresh your memory of how to add fractions without using a calculator! The skill of adding fractions without a calculator is necessary when the fractions involve algebra.

A guide on how to add fractions can be found here:

- https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/fractions-unlike-denom-pre-alg/v/adding-fractions-with-10-and-100-as-denominators

This topic also builds on the fundamental concepts of algebra, so make sure you've seen **Introduction to Algebra** before watching this video.

## The Video Content

This topic explores how to add algebraic fractions. Algebraic fractions are added in the same way as numerical fractions.

Here is a sample question.
Simplify:

$$3k \,/\, 5 + k \,/\, 2$$

What does it mean by 'simplify'?

**Step 1 Understand the question**

Simplifying an expression means to write it (express it) as a single fraction.

**Step 2 Develop a plan**

How do we add fractions? When adding numerical fractions, we make the numbers on the bottom the same, then add the numbers on the top. In mathematical terms, we call the number on the bottom a denominator, and the number on the top a numerator. It's the same with algebraic fractions: we have to find a common denominator. So this is our plan.

**Step 3 Carry out the plan**

To simplify:

$3k$ / 5 + $k$ / 2

we must find a common denominator. So we need a number that can be divided by both 5 and 2. In this case 10 will work.

For $3k$ / 5 – the first term – 5 has to be multiplied by 2 to get 10. So we multiply both the top and the bottom by 2. In other words, we multiply by 2 / 2. The fraction 2 / 2 is, of course, just 1, so we are really multiplying by 1, which isn't changing the number at all.

For $k$ / 2 – the second term – 2 has to be multiplied by 5 to get 10. So we multiply the top and bottom by 5.

When we multiply fractions we multiply top and bottom. For the first term, $3k$ / 5:

| | |
|---|---|
| top | $3k$ x 2 = $6k$ |
| bottom | 5 x 2 = 10 |

So our first term becomes $6k$ / 10.

The second term, using the same method, becomes $5k$ / 10. So, to summarise:

$3k$ / 5 + $k$ / 2

= $3k$ / 5 x 2 / 2 + $k$ / 2 x 5 / 5

= $6k$ / 10 + $5k$ / 10

Remember, this expression is exactly the same as the first one, because all we did was multiply by 1. But now the denominators are the same, we can add the numerators:

$6k$ / 10 + $5k$ / 10 = ($6k$ + $5k$) /10

$6k$ + $5k$ is simply $11k$

The final answer is:

($6k$ + $5k$) / 10 =$11k$ / 10

We can't simplify that further, because we don't know what $k$ is, so we're done.

**Another example**

What if the letters are on the bottom? The approach is the same. Let's try simplifying:

5 / $a$ + 7 / $3b$

Find a denominator that both "$a$" and "$3b$" go into. Then multiply both of them together and get $3ab$.

$a$ x $3b$ = $3ab$

Here's a hot tip: if you are struggling to find a common denominator, multiplying the denominators together will always work.

$a$ goes into $3ab$ $3b$ times, so we multiply top and bottom of the first part by $3b$.
$3b$ goes into $3ab$ $a$ times, so we multiply top and bottom of the second part by $a$:

5 / $a$ + 7 / $3b$

= 5 / $a$ x $3b$ / $3b$ + 7 / $3b$ x $a$ / $a$

and multiply to get

= $15b$ / $3ab$ + $7a$ / $3ab$

Now we have a common denominator. Next put the numerators all over the same denominator:

($15b$ + $7a$) / $3ab$

Done!

## *Some Practice Questions*

Simplify the following:

1. $x / 3 + x / 2$
2. $m / 4 + m / 7$
3. $3 / t + t / 3$
4. $a / b + c / d$
5. $5 / (x + 1) + 3 / (x + 1)$
6. $1 / p^3 + 2 / k^2$
7. $(k + 2) / (x + 1) + 3 / (x - 1)$
8. $7 / 4w + 2 / 5wr$

Answers

1. $5x / 6$
2. $11m / 28$
3. $(9 + t^2) / 3t$
4. $(ad + bc) / bd$
5. $8 / (x + 1)$
6. $(k^2 + 2p^3) / k^2p^3$
7. $(kx - k + 5x + 1) / (x^2 - 1)$
8. $(35r + 8) / 20wr$

Take a look at the working out for each answer **here**.

## Now What?

This video introduces working with algebraic fractions, but only deals with addition.

However, subtraction and addition of fractions with algebra are very similar to subtraction and addition of fractions with common, everyday numbers. Multiplication and division of algebraic fractions are also similar to fractions with common, everyday numbers: multiplication of algebraic fractions is done by multiplying the denominators and multiplying the numerators, and division is done by the process of inverse and multiply.

When you're confident with adding algebraic fractions, the next step is to become familiar with subtraction, multiplication and division. Again, this is best done by refreshing your memory of how to subtract, multiply and divide with common, everyday numbers first, then moving onto algebraic fractions after that.

To practise adding, subtracting, multiplying and dividing fractions, see the Other Links section below.

## But When Am I Going To Use This?

Just like fractions with everyday numbers, fractions with algebra appear in countless different situations. What if you're splitting the bill of "$m$" dollars between "$n$" people? Then the cost per person is $m$ / $n$. The old formula for gradient, $m$ = rise / run is an algebraic fraction. When you pay off a home loan you must divide the total cost over "$n$" number of months, creating another fraction. Any time you are dividing by an unknown quantity (represented by a letter), or dividing an unknown quantity, you are creating an algebraic fraction.

## Other Links

**Sophia.org** gives several different tutorial videos for each concept, and also has as a quiz so you can test yourself.

- https://www.sophia.org/concepts/algebraic-fractions-with-unlike-denominators

**Mathspace** has several worked video examples for addition and subtraction of algebraic fractions, as well as multiplication and division of algebraic fractions. Mathspace is also available on the App Store, Google Play or Microsoft Store. It has questions to test yourself, however, signing up is required to access those features.

- https://mathspace.co/learn/world-of-maths/algebra/rational-expressions-addsub-unlike-denominators-24309/adding-subtracting-algebraic-fractions-953/

**Maths is Fun** provides simple and well-written summaries of the rules, worked examples, and a small selection of sample questions. This page gives a summary of addition, subtraction, multiplication and division, and also explains each operation separately.

- https://www.mathsisfun.com/algebra/fractions-algebra.html

# Components of Vectors

## Before You Watch

This video builds directly on the concepts discussed in Introduction to Trigonometry, which presented the three basic trigonometric ratios: sin, cos and tan. If you haven't watched the **Introduction to Trigonometry** video yet, do that first, then come back.

## The Video Content

Think about a ball being thrown. The ball moves up as you throw it, and it also moves across the field. To predict the path of the ball, we need to break up the movement into two parts: the vertical component, up and down, and the horizontal component, across the field.

This is called breaking down the movement into perpendicular components. It is used all the time in engineering and physics and is an excellent example of a common use of trigonometry.

**Step 1 Understand the question**

Let's say that you throw a ball at 120 km/hr, at an angle of 22$^\circ$ above the horizontal. The question is: what are the vertical and horizontal components of this initial motion?

**Step 2 Develop a plan**

Our starting throw has a length of 120 and is 22$^\circ$ above the horizontal.

These two lines, one horizontal and one vertical, are called the vertical and horizontal components of our starting throw. We need to find these two lengths, $x$ (horizontal) and $y$ (vertical).

That's the plan.

**Step 3 Carry out the plan**

Remember in trigonometry, the sine of an angle is equal to the length of the opposite side over the length of the hypotenuse.

So:

$\sin 22^\circ$ = opposite / hypotenuse

= $y$ / 120

To get the $y$ on its own, multiply both sides by 120 and cross off the 120 top and bottom on the right.

This gives us:

$120 \sin 22^\circ = y$

Similarly for the horizontal, cosine is the adjacent side over the hypotenuse.

Therefore:

$\cos(22^\circ) = x / 120$

As before, multiply both sides by 120, cross off 120 top and bottom and we have:

$120 \cos(22^\circ) = x$

Now we put this into the calculator and get:

$y = 44.95$ (to 2 decimal places)

and

$x = 111.26$ (to 2 decimal places)

So, our initial throw of 120 km/hr consists of moving across the field at 111.26 km/hr and vertically up at 44.95 km/hr. Of course, gravity will pull the ball down, so this will change over time.

### *Did you know?*

Whenever doing trigonometry, it is very important to check that your calculator is in the correct mode, depending on what the angles are being measured in. In this example, we are measuring the angles in degrees, so we must make sure that our calculator is in degrees mode. The next most common way of measuring angles is in Radians, but for this example it's important that our calculator is not in Radians mode. If it is, our answer will be wrong.

**Step 4 Reality Check**

Do these numbers make sense? At a glance, the numbers seem ok; neither of them seems too big, nor too small to be unreasonable. If we wanted to be very sure, we could check using Pythagoras' theorem. Give it a go and see if it adds up!

Put our two answers into Pythagoras' theorem:

$$\sqrt{(44.95^2 + 111.26^2)}$$

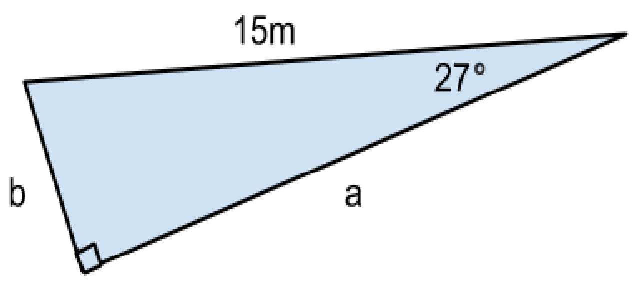
Put this into the calculator and we get:

$$\sqrt{(44.95^2 + 111.26^2)} = 119.9970 \text{ (to 4 decimal places)}$$

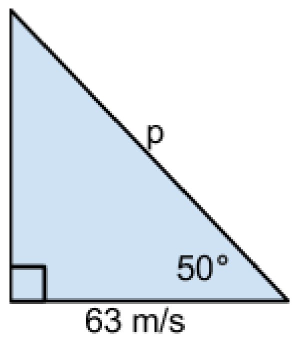
We don't expect to get *exactly* our original speed of 120, because we rounded off our answers to 2 decimal places. However, the answer is very close to 120, so we can be more confident that our answer is reasonable.

## *Some Practice Questions*

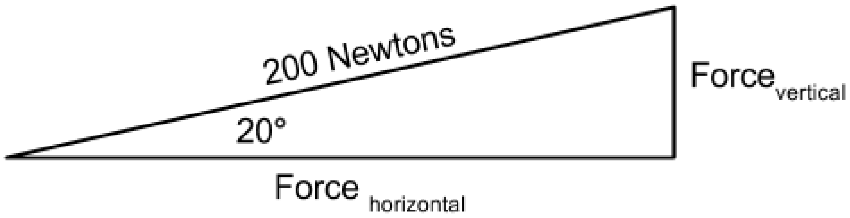
1. Find the lengths of the sides $a$ and $b$, of the following triangle:



2. Find the length of the side $p$ of the following triangle:




3. Consider a box is being pulled by a rope with a force of 200 Newtons. The rope is angled 25° above the horizontal. In other words, the force acting on the box can be visualised by the triangle below. What is the horizontal component of this force?




4. If a bullet is fired at 400 m/s at 10° above the horizontal, what is the vertical component of its velocity?

Answers

1. a = 13.37m, b = 6.81m (2 decimal places)

2. 98.01 m/s (2 decimal places)

3. 187.94 N (2 decimal places)

4. 69.46 m/s (2 decimal places)

Take a look at the working out for each answer **here**.

## Now What?

After watching this video, plus **Introduction to Trigonometry**, you should be comfortable with finding the length of a side of a right-angled triangle when given the length of another side and an angle of the triangle.

Beyond this, it is recommended that you investigate other applications of trigonometry, such as finding an angle when given the sides of the triangle, or the sine and cosine rules. You can explore these through websites such as https://www.khanacademy.org/math/trigonometry/basic-trigonometry.

Alternatively, you can learn about other algebraic concepts such as **Algebraic Fractions** or **Factorisation of Algebraic Expressions**.

## But When Am I Going To Use This?

The obvious application of trigonometry is for right-angled triangles. This is common in construction, surveying and the application discussed in this video, where things like forces, or movement, are broken down into perpendicular components. However, trigonometry can also extend to non-right angled triangles through, for example, the sine rule and the cosine rule.

Moving beyond triangles, trigonometry is critical to the study of waves, such as radio waves. This is very important in fields such as wireless communication and quantum physics: for instance, mobile phone technology would be impossible without a method of breaking down a signal into a series of sines and cosines known as Fourier Analysis.

## *Other Links*

**Maths is Fun** has a great page summarising the trigonometric ratios and includes several different applets to help you visualise and understand the basics. It also has other pages dealing with more advanced trigonometry subjects to develop your understanding.

- https://www.mathsisfun.com/algebra/trigonometry.html

The **Mathspace** page on trigonometry is well laid out with a variety of instructive videos, applets and quizzes. It also has apps for iPad, iPhone, Android and Windows Phone. It does require you to create a login, however, using these resources is free.

- https://mathspace.co/learn/world-of-maths/trigonometry/trigonometric-ratios-6831/special-ratios-258/

**GeoGebra** is a mathematics app that works on a wide variety of platforms, including tablets, and in a web browser. It is used to create great tools for teaching and learning. The link here is an example of a GeoGebra 'program' that helps demonstrate how trigonometric ratios stay constant, no matter how the triangle shrinks and expands.

- http://www.geogebra.org/student/b77950#material/11887

The following link is to a simple animation which demonstrates the creation of the graph of the sine function using the unit circle. The source code for the creation of the animation is also attached. For people with a natural orientation towards computer programming (which is a huge application of mathematics) this may help you understand the nature of trigonometry.

- http://www.humblesoftware.com/demos/trig

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the chapter covering basic trigonometry, and continues onto more advanced trigonometric concepts.

- https://www.khanacademy.org/math/trigonometry/basic-trigonometry

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. This video covers the trigonometric ratios and solves a variety of problems using these ratios.

- http://patrickjmt.com/right-triangles-and-trigonometry/

# Differentiation of Polynomials

## Before You Watch

This video continues directly on from **Rates of Change and Differentiation**, in which the concept of a rate of change was introduced and we investigated the need to know the value of instantaneous rates of change. The term 'to differentiate' was also discussed. Differentiation is the process of calculating a rate of change. This video describes how to differentiate a specific class of equations known as polynomials. So make sure you've seen **Rates of Change and Differentiation** recently before watching this video.

This topic also builds upon earlier algebraic concepts such as **indices**, including **negative indices**, and **linear equations**. It is important to be comfortable with algebra and manipulating algebraic equations before continuing with calculus, so watch those videos again if you need to, then come back.

## The Video Content

This topic builds on the previous calculus videos, and looks at how to differentiate a polynomial.

Let's say a car's position $p$ is given by $3t^2$. So:

$$p = 3t^2$$

How do we calculate the speed?

**Step 1 Understand the question**

As we saw from the last video, to calculate the speed of a car, we need to calculate the rate of change between position and time.

**Step 2 Develop a plan**

To calculate the rate of change, we need to differentiate the equation. That's the plan.

**Step 3 Carry out the plan**

How to find the derivative of an equation depends on what kind of equation is it.
The equation $p = 3t^2$ is a polynomial.

Here are some examples of polynomials:

- $p = 3t^2$
- $j = 6r + 3r^2$
- $m = n^3$
- $y = 2x^2 + 7x + 3$

They have two different letters. One letter is on its own on one side of the equation and all the terms on the other side have the other letter, or just a number. The terms on the other side can have a number in front of the letter, and they can also have any integer power.

***Did you know?***

For an equation to be a polynomial, the power must be a positive integer. If there are any negative powers then it is not a polynomial.

To differentiate a polynomial, first we'll look at what happens to the left hand side.

As we saw in the last topic, differentiating calculates the rate of change. So instead of just position, we have change in position over change in time. In other words:

$$dp \,/\, dt$$

Now we can differentiate the right hand side of the equation.

To do this, take the power and bring it out the front to multiply our expression with. The 3 and the $t$ are left alone. Subtract 1 from the power, so the power changes from 2 down to 1:

$$p = 3t^2$$
$$dp \,/\, dt = 2 \text{ x } 3t^1$$

We can then simplify that, so we get:

$dp$ / $dt$ = 6$t$

That's our answer. The speed of the car is given by $dp$ / $dt$ = 6$t$

Let's do another example.

This time we will differentiate:

$y = 4x^7 + 2x^3 + x$

The process is the same as before. The letter on its own is now $y$, and the letter on the other side is $x$.

So when we differentiate the $y$ becomes $dy$ / $dx$.

On the right hand side we deal with one term at a time.

Take the 7 down, leave the $4x$ as is, and reduce the power by 1, down to 6.

For the next one we pull the 3 down, leave the 2x, and reduce the power by 1.

For the last one we remember that $x$ means $x$ to the power of 1, so we pull the 1 out the front, and reduce the power to 0.

From the indices video we saw that anything to the power of zero is 1, so our last term just becomes 1:

$y = 4x^7 + 3x^3 + x$

$dy$ / $dx$ = 7 x $4x^7$ + 3 x $2x^2$ + 1 x $x^0$

Simplifying, we see our answer is:

$dy$ / $dx$ = $28x^6 + 6x^2 + 1$

Done! You have found the derivative of a polynomial.

## Some Practice Questions

Differentiate:

1. $y = x^2$
2. $p = 4t^2 - 3t$
3. $j = 5r^6 - 7r^4 + 5r^2$
4. $q = (7x + 3x^2)^2$ (Hint: expand out the brackets first)

Answers

1. $dy / dx = 2x$
2. $dp / dt = 8t - 3t$
3. $dj / dr = 30r^5 - 28r^3 + 10r$
4. $dq / dx = 36x^3 + 126x^2 + 98x$

Take a look at the working out for each answer **here**.

## Now What?

By now you will be familiar with the three videos that introduce the differential branch of calculus: **Introduction to Calculus**, **Rates of Change and Differentiation** and this topic, **Differentiation of Polynomials**. You should understand the core concepts of calculus and know what a rate of change is. You will also know that differentiation is all about calculating the rate of change, and know how to differentiate one category of equations, the polynomials. From here there are two main directions you can go.

One option is to explore how to differentiate other types of equations, such as those involving trigonometry, or exponentials. To do this you should consider looking at sites such as the Khan Academy at https://www.khanacademy.org/math/differential-calculus/taking-derivatives

Alternatively, you could investigate the other branch of calculus, integral calculus. This is introduced in the topic **Integration**.

## But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. For instance, velocity (or speed) is a change of position over a change in time, and acceleration is a change in velocity over a change in time – so any motion is studied using calculus. Other examples include the flow of water through pipes over time, or changing commodity prices against demand. Because change is everywhere, the potential applications for calculus are endless, particularly in engineering and science. Calculus is necessary knowledge for any degree related to engineering or science.

## Other Links

**Maths is Fun** has a great page that takes you through a simple problem which highlights the need for calculus to discuss changes happening around us. It then continues to explore the main two areas of calculus, differentiation and integration, and provides regular questions to test your understanding.

- https://www.mathsisfun.com/calculus/introduction.html

**IntMath** gives a bit of historical perspective to explain the sometimes confusing notation that is used in calculus, discussing how it is the mixed product of two mathematicians working independently. It also provides some excellent examples of applications of calculus that are in common use today, as well as helpful applets to understand both differential and integral calculus.

- http://www.intmath.com/calculus/calculus-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical and other concepts, as well as questions to test your knowledge. This content provides a whole chapter on taking the derivatives, including of harder equations not covered in this video.

- https://www.khanacademy.org/math/differential-calculus/taking-derivatives

**Patrick JMT** (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This content runs through differentiation of simple polynomials, but the site also provides videos demonstrating more complex differentiation.

- http://patrickjmt.com/basic-derivative-examples/

# Exponentials and Logarithms

## Before You Watch

This topic introduces solving equations where the letter you are asked to find the value of is in the index (also called the exponent). This requires the use of logarithms. The definition of a logarithm is:

if:

$$a^{b} = c$$

then:

$$\log_{a}c = b$$

When we read the line above, we say “log to the base ‘$a$’ of ‘$c$’, is equal to '$b$'”.

Another way we can think about logarithms is as the inverse of the exponential. For example, if we have a number $x$, and we put a number, say, 5 to the power of $x$:

$$5^{x}$$

then we put that to the log of base 5:

$$\log_{5}(5^{x})$$

that is equal to $x$:

$$\log_{5}(5^{x}) = x$$

In other words, if we start with $x$, and then put 5 to the power of $x$, we can think of the log process as “undoing” the process of putting 5 to the power of $x$. So these two processes are the inverse of each other, in a similar way to multiplication and division being the inverse of each other (that is, if you start with $x$, then multiply by 5, then divide by 5, you get back to $x$).

Need more of an introduction to the nature of logs? Look at one of the links below before watching this video.

- https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms
- https://www.mathsisfun.com/algebra/logarithms.html

As well as having an understanding of the nature of logs, it is useful to revise your knowledge of the number "$e$" before watching this video. Remember, "$e$" is an irrational number that has many real world applications, and that's why it has been given a special name. In fact, "$e$" is similar to the number $\pi$, which is also an irrational number that has a special name because of its applications. The number "$e$" is approximately equal to 2.718, however, most calculators have a dedicated button for it.

Comfortable with using logarithms and the number "$e$"? Then you're ready for this topic!

## The Video Content

This content explores how to solve equations using logarithms. You might recall from high school that just as division is the inverse of multiplication, logs are the inverse of exponentials: for example, $2^5$ is an exponential. Logarithms are very useful in mathematics. A very important property of logarithms is that the log of some number – let's call it $r$ – to the power of some other number – say $p$ – is equal to $p$ times the log of $r$. For example:

$$\log(7^5) = 5 \log(7)$$

Say we are asked to solve:

$$5^p = 16$$

**Step 1 Understand the question**

What the question is really asking us to do is rearrange this equation and have $p$ on its own on one side, with what it equals on the other.

5 to the power of $p$ equals 16, so we need to find the value of $p$ such that 5 to the power of that value gives 16. We already know 5 to the power 2 = 25 and 5 to the power 1 = 5, so $p$ must be somewhere between 1 and 2.

Here's a hot tip: doing this kind of quick estimate in your head can be a very effective way of catching mistakes!

## Step 2 Develop a plan

Remember that we always do the same to both sides of an equals sign. Here we need to take the log of both sides, solving the equation by using the properties of logs. That's the plan.

## Step 3 Carry out the plan

Taking the log of both sides gives us:

$$\log(5^p) = \log(16)$$

Using the property from the start of the topic, we know:

$$\log(5^p) = p \times \log(5)$$

So we have:

$$p \times \log(5) = \log(16)$$

Divide both sides by log 5 and we get:

$$p = \log(16) / \log(5)$$

Now we have something we can enter into a calculator. It doesn't matter which base we use, either base 10 or base $e$, it'll give the same answer regardless. In this case $p$ is approximately 1.7 or, with more decimal places, 1.7227.

## Step 4 Reality check

We know that $p$ should lie between 1 and 2, so the answer already seems reasonable. To make sure it is correct, we can take the answer, $p = 1.7227$, and put it back into the equation:

$$5^p = 16$$

and see if the left and right sides of the equation match.

It won't match exactly since we used an approximation for $p$, but it should be close. Give it a try. See if:

$$5^{1.7227} = 16$$

Putting $5^{1.7227}$ into the calculator gives a result of:

$$5^{1.7227} = 15.9998$$

rounded off to 4 decimal places.

Obviously this isn't exactly the answer 16 from our question, but remember that 1.7227 was already rounded off from what was on the calculator, therefore we don't expect the answer to be exactly 16.

The number we got, however, is so close to 16 (in fact, it is 16, rounded off to three decimal places) that we can be confident this is the correct answer.

## Some Practice Questions

1. Solve for $m$ $2^m = 8$
2. Solve for $u$ $5^u = 78{,}125$
3. Solve for $t$ $e^t = 15$
4. Solve for $n$ $4^n = \pi$

Answers

1. $m = 3$
2. $u = 7$
3. $t = 2.708$ (to 3 decimal places)
4. $n = 0.826$ (to 3 decimal places)

Take a look at the working out for each answer **here**.

## Now What?

This topic explains how to solve an important category of equations, where the letter is in an index. It will also have refreshed your memory of logarithms and the number “$e$”.

Once you are confident in your ability to solve equations like this, check your skill at solving other types of equations, such as **Quadratic Equations** or **Simultaneous Equations**.

## But When Am I Going To Use This?

Exponentials are often found in situations where the rate that something grows is related to the size it already is. A very simple example of this is a home loan: the bigger the loan or interest rate, the faster it grows. This type of growth is found naturally as well, for instance, in bacterial growth.

Logarithms are also commonly used to measure things over a very large scale, where the values getting measured can be very large and very small. For example, the decibel, the standard unit for measuring the volume of sounds, is a logarithmic scale. Another example of a logarithmic scale is the Richter scale, the standard scale for the measurement of the strength of earthquakes.

## Other Links

**Fort Bend Tutoring** features a series of YouTube videos covering many different mathematical concepts. The video below covers solving exponential equations, and also demonstrates how to solve exponential equations in situations where you don’t need to use the logarithms.

- https://www.youtube.com/watch?v=Y-PaFgFDLZk

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. Look at the top of this resource for the link to the chapter that covers logarithms. Half way through the chapter is a video that shows you how to solve the exponential equation.

- https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/exponential-equation

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. The video linked below demonstrates how to solve exponential equations.

- http://patrickjmt.com/solving-exponential-equations/

# Introduction to Trigonometry

## Before You Watch

This topic is designed to refresh your memory of trigonometry and what it means. It uses algebra to represent the various sides and angles of the triangle. So, if you're not confident with the concept of algebra and how symbols represent certain values, watch **Introduction to Algebra** first, then come back.

## The Video Content

Trigonometry is a branch of mathematics that studies relationships involving lengths and angles of triangles. It first came about when people were trying to figure out distances between stars, more than 2000 years ago. Astronomers noted that when the length of one side and the value of one angle is known, then all other angles and lengths can be determined.

The Greeks worked out what we now know as trigonometric ratios: sine, cosine and tangent, which you probably studied in high school. Remember SOH-CAH-TOA?

SOH-CAH-TOA is a trick to remember the ratios.

It is important to understand that a ratio is a relationship between two numbers. In this case the numbers are the length of the sides of the triangle.

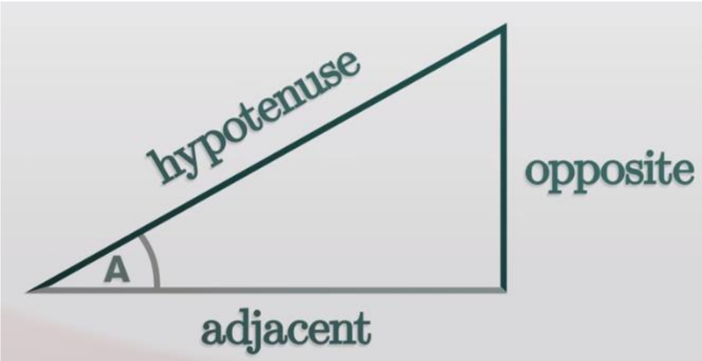
If we have a right triangle and a given angle A in it, we can label the sides of the triangle as follows:

- the longest side is always the **hypotenuse**
- the side that is next to the angle is called **adjacent** side
- the other side is called the **opposite** side.

The sine of A is the length of the opposite side divided by the length of the hypotenuse:

sin A = opposite / hypotenuse

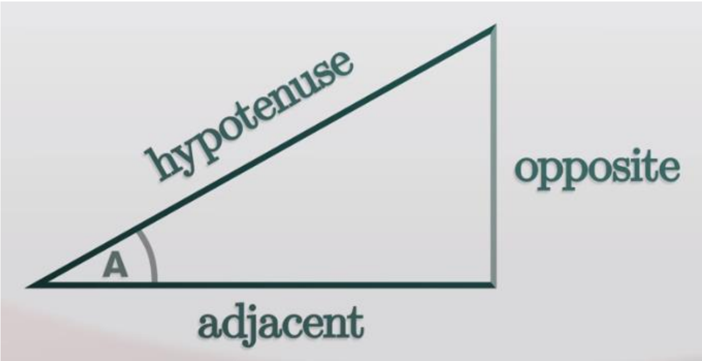
This is **SOH**.




The cosine of A is the length of the adjacent side divided by the length of the hypotenuse:

cos A= adjacent / hypotenuse

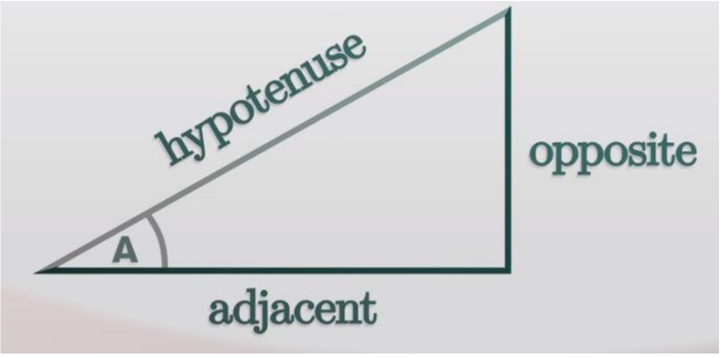
This is **CAH**.




The last one is the tangent of A. The tangent of A is the length of the opposite side divided by the length of the adjacent side:

tan A = opposite / adjacent

This is TOA.




So that's SOH-CAH-TOA.

These ratios always remain constant, no matter how big or small the right triangle is, as long as the angle A doesn't change.

This property of triangles is incredibly useful and is even what allows your GPS to always find where you are and give you the quickest way to your destination.

## Now What?

The next step in your trigonometry journey is to watch **Components of Vectors**. This video demonstrates a specific application of trigonometry used in science and engineering, as well as giving sample questions that you're likely to come across at university and beyond.

## But When Am I Going To Use This?

Right-angled triangles are common in construction, surveying and the application discussed in the **Components of Vectors** video, where things like forces, or movements, are broken down into perpendicular components. However, trigonometry also extends to non-right angled triangles through applications like the sine rule and the cosine rule, and to calculating the areas of triangles.

Moving beyond triangles, trigonometry is critical to the study of waves, such as radio waves. This is very important in fields such as wireless communication and quantum physics: for instance, mobile phone technology would be impossible without a method of breaking down a signal into a series of sines and cosines known as Fourier Analysis. Your GPS device also uses trigonometry to help you navigate using satellites.

## *Other Links*

**Maths is Fun** has a great page summarising the trigonometric ratios and includes several different applets to help you visualise and understand the basics. It also has other pages dealing with more advanced trigonometry subjects to develop your understanding.

- https://www.mathsisfun.com/algebra/trigonometry.html

The **Mathspace** page on trigonometry is well laid out with a variety of instructive videos, applets and quizzes. It also has apps for iPad, iPhone, Android and Windows Phone. It does require you to create a login, however, using these resources is free.

- https://mathspace.co/learn/world-of-maths/trigonometry/trigonometric-ratios-6831/special-ratios-258/

**GeoGebra** is a mathematics app that works on a wide variety of platforms, including tablets, and in a web browser. It is used to create great tools for teaching and learning. The link here is an example of a GeoGebra 'program' that helps demonstrate how trigonometric ratios stay constant, no matter how the triangle shrinks and expands.

- http://www.geogebra.org/student/b77950#material/11887

The following link is to a simple animation which demonstrates the creation of the graph of the sine function using the unit circle. The source code for the creation of the animation is also attached. For people with a natural orientation towards computer programming (which is a huge application of mathematics) this may help you understand the nature of trigonometry.

- http://www.humblesoftware.com/demos/trig

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link takes you to the chapter covering basic trigonometry, and continues onto more advanced trigonometric concepts.
https://www.khanacademy.org/math/trigonometry/basic-trigonometry

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. This video covers the trigonometric ratios and solves a variety of problems using these ratios.

- http://patrickjmt.com/right-triangles-and-trigonometry/

# Negative and Fractional Indices

## Before You Watch

This topic builds directly on the work done in Indices Laws, and also introduces new laws. Make sure you are familiar with the following rules of indices before starting this topic:

- $a^n \times a^m = a^{(n+m)}$
- $a^n / a^m = a^{(n-m)}$
- $(a^n)^m = a^{(n \times m)}$

Confident in the use of these laws of indices? Then you're ready! If not, watch **Indices Laws** first, then come back.

## The Video Content

This content explains the mathematical meaning of indices, or powers, that are either zero, negative, or a fraction. It builds on the previous topic, so if anything doesn't make sense, refer to that video, then return.

To understand the meaning of something to the power of zero, let's consider:

$$t^2 / t^2$$

Using our index laws, we know this is:

$$t^2 / t^2 = t^{(2-2)}$$

which is:

$$t^{(2-2)} = t^0$$

We also know it is equal to 1, because anything divided by itself is equal to 1:

$$t^2 / t^2 = 1$$

Therefore:

$$t^0 = 1$$

So, anything to the power of 0 is 1.

Let's look at **negative indices**. A negative power means one over the positive power. For example:

$$p^{-6} = 1 / p^6$$

Consider:

$$f^3 \times f^{-2}$$

This can be solved in two ways. We know when we multiply letters with powers, we add the powers:

$$f^3 \times f^{-2} = f^{(3+(-2))}$$

Therefore:

$$f^3 \times f^{-2} = f^{(3-2)} = f$$

Another way is to use our new meaning of negative powers. We know that:

$$f^{-2} = 1 / f^2$$

Therefore:

$$f^3 \times f^{-2} = f^3 \times 1 / f^2$$

which is:

$$f^3 \times 1 / f^2 = f^3 / f^2$$

When dividing letters with powers, we subtract the powers, so this is:

$$f^3 / f^2 = f^{(3-2)}$$

which is:

$$f^{(3-2)} = f$$

Either way we get the same answer.

Now let's look at **fractional indices**.

A fractional index of $1/n$ is the same as taking the $n$th root. For example:

$$y^{1/3} = \sqrt[3]{y}$$

### *Did you know?*

This section on fractional indices applies to fractional indices of positive numbers. It is not so straight-forward for negative numbers. Try taking your calculator and finding:

$$\sqrt{-1} = (-1)^{½}$$

You should get an error. This is a branch of mathematics called complex numbers, or imaginary numbers.

You may come across complex numbers later on, but for now just keep in mind that this section works for positive numbers only.

Another example:

$$(d^2)^{1/2}$$

Here, we could use the index laws for powers of powers. This means:

$$(d^2)^{1/2} = d^{2 \text{ x } 1/2}$$

which is:

$$d^{2 \text{ x } ½} = d$$

Or we could use the law for fractional indices, so:

$$(d^2)^{1/2} = \sqrt{d^2}$$

which is just:

$$\sqrt{d^2} = d$$

Once again, we get the same answer using either method.

Consider:

$$w^{3/4}$$

Using indices laws this is the same as:

$$w^{3/4} = (w^3)^{¼}$$

or in other words:

$$(w^3)^{¼} = \sqrt[4]{w^3}$$

## *Some Practice Questions*

Convert these expressions to terms using just fractions and root symbols – that is, without using negative or fractional indices.

1. $a^3 / a^6 =$

2. $3^{-1}b^5 \times 3^2 b^{-6} =$

3. $9^{1/2}m^{1/3} =$

4. $(k^{4/3})^{1/2} =$

Convert these expressions to terms using just negative and fractional indices – that is, without using fractions or root symbols.

5. $\sqrt[3]{k} =$

6. $\sqrt[4]{y} / y^4 =$

7. $40 / (2h)^2 =$

8. $1 / \sqrt{(5p)}^4 =$

Answers

1. $1 / a^3$

2. $3 / b$

3. $3\sqrt[3]{m}$

4. $\sqrt[3]{k^2}$

5. $k^{1/3}$

6. $y^{-3\frac{3}{4}}$

7. $10h^{-2}$

8. $0.04p^{-2}$

Take a look at the working out for each answer **here**.

## Now What?

This video, along with Indices Laws, covers important concepts in algebra that you will continue to run into throughout mathematics, such as in calculus and more advanced areas. So it's very important that you're familiar with these rules and comfortable using them.

Once you are confident with these rules, why not check your skills in some of the areas covered by the other algebra videos? Look at, for instance, **Factorisation of Algebraic Expressions** or **Algebraic Fractions**.

## But When Am I Going To Use This?

Indices are used in many different situations in real life. A common example is in writing very large or very small numbers. These are often written in scientific notation, and can be stored in computers as a type of variable known as a floating point variable. Scientific notation makes heavy use of indices to keep numbers easier to work with. Floating point variables are very important in all areas of computing, including gaming physics.

Indices are also used in the calculation of areas and volumes. For example, the area of a square is the length squared, and the volume of a cube is the length cubed. This is especially important when changing units of measurement, such as from cubic metres to cubic centimetres.

Plus, indices are used in certain kinds of other measurements, including acidity (pH), the loudness of sound (decibels), or the intensity of earthquakes (the Richter scale). All of these measurements use what is known as a logarithmic scale, which relies on indices.

## Other Links

**Maths is Fun** has useful applets to help you understand the basic idea of indices, plus an easy to follow summary of the rules. The first link below is the same as for the topic Indices Laws and also covers negative indices and an index of zero. The next two links cover negative and fractional indices respectively. Sample questions are provided.

- http://www.mathsisfun.com/algebra/exponent-laws.html
- http://www.mathsisfun.com/algebra/negative-exponents.html
- http://www.mathsisfun.com/algebra/exponent-fractional.html

**Laerd Mathematics** gives a succinct summary of the rules, and follows this up with a wide selection of questions with worked answers available.

- http://mathematics.laerd.com/maths/indices-intro.php

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Here are two relevant videos: the first runs through the use of negative indices; the second explains fractional exponents. On both of these pages are links below the video to more videos covering example questions.

- http://patrickjmt.com/negative-exponents/
- http://patrickjmt.com/evaluating-numbers-raised-to-fractional-exponents/

# Quadratic Equations

## Before You Watch

This topic covers how to solve a particular type of equation called a quadratic equation.

To get the most out of this content, you'll need to be comfortable with the rearrangement of algebraic equations, as well as indices as they are applied to algebra. To brush up on your skills in these areas, take a look at the **Introduction to Algebra** and **Indices Laws** videos first, then come back.

## The Video Content

You will often see equations that look like this:

- $2x^2 - 1 = -x$
- $k^2 + 6k + 8 = 10$
- $p^2 - 2p = 0$

These equations are common in the real world. They help us figure out things like the side length of a square when we are given its area, or the trajectory of a ball when it is thrown.

These equations are called quadratic equations and they only have one unknown. In the above examples the unknowns are $x$, $k$ and $p$. When they are expanded, they have a squared term in their unknowns but don't have any powers higher than 2.

Let's look at the first example.

Solve the equation:

$$2x^2 - 1 = -x$$

So, what is the question asking?

**Step 1 Understand the question**

When we are asked to solve a quadratic equation, we simply have to find the value of $x$ which makes the equation true. Several methods can be used to do this, however, the most reliable is the quadratic formula. Therefore, using the formula is our plan!

***Did you know?***

Another method of solving quadratics often taught in schools is to move all the terms onto one side of the equation, so only a 0 is left on the other side, then factorise the equation. While this method works sometimes, and helps with understanding quadratic equations, it is unreliable. That is, it is often very difficult to see the correct way to factorise a quadratic equation, unless the question has been crafted to make the factorisation easy. For this reason, in practise most mathematicians will simply go straight to the quadratic formula, the strategy we are using here.

**Step 2 Develop a plan**

What is the quadratic formula?

The quadratic formula is a rule that allows us to solve any quadratic equation.

This equation is in its general form:

$$ax^2 + bx + c = 0$$

The quadratic formula looks like this:

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$$

Now we can carry out the plan by using the formula.

**Step 3 Carry out the plan**

First, to make sure the equation is in the general form, put everything on one side of the equals sign, leaving just a zero on the other side.

In our example:

$$2x^2 - 1 = -x$$

the easiest way to do that is to add $x$ to both sides:

$$2x^2 - 1 + x = -x + x$$

giving us:

$$2x^2 - 1 + x = 0$$

Next, order the terms in decreasing order of the power of the letter. That is, put the $x$ squared term first, then the $x$ term, then the constant. Then compare this equation to the general form of the quadratic equation. To do this, write one above the other:

$$2x^2 + x - 1 = 0$$

$$ax^2 + bx + c = 0$$

By comparing these two equations we can see that:

$a$ is equal to 2

$b$ is equal to 1

$c$ is equal to -1

Now we substitute these values into the quadratic formula, which is:

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$$

Since we know that:

$a = 2$

$b = 1$

$c = -1$

we can write:

$$x = (-1 \pm \sqrt{(1^2 - 4 \times 2 \times (-1))}) / 2 \times 2$$

which equals:

$$x = (-1 \pm \sqrt{(1 - (-8))}) / 4$$

The double negative becomes a positive, and so adding these terms gives:

$$x = (-1 \pm \sqrt{9}) / 4$$

The only step left is to produce the solutions. Since the square root of 9 is 3, the quadratic formula gives us:

$$x = (-1 \pm 3) / 4$$

Because of the ± sign, we have two possible solutions – one when we add 3 and another when we subtract it:

$x = (-1 + 3) / 4$ or $x = (-1 - 3) / 4$

$x = 2 / 4$ or $x = -4 / 4$

$x = ½$ or $x = -1$

That's how we arrive at the two solutions: $x = 1 / 2$ or $x = -1$.

**Step 4 Reality check**

To check that these answers are reasonable, substitute these values into the original equation:

$$2x^2 - 1 = -x$$

Does the left side of the equation equal the right side? Try it!

First, substitute $x = ½$ into the above equation:

$$2(½)^2 - 1 = -½$$

Then simplify it:

$2(¼) - 1 = -½$

$½ - 1 = -½$

$-½ = -½$

The last line is obviously true; the left side equals the right side, therefore we know that ½ is a correct solution.

Second, substitute $x = -1$ into the equation:

$$2(-1)^2 - 1 = -(-1)$$

Then simplify it:

$2(1) - 1 = 1$

$2 - 1 = 1$

$1 = 1$

Once again, the left and right sides of the equations match up, which confirms that the other solution is $x = -1$.

Quadratic equations in general have two solutions, but they cannot have more than two solutions. It is, however, possible to have only one solution, when the part inside the square root symbol of the quadratic formula is equal to zero.

## Some Practice Questions

1. $k^2 + 6k + 8 = 10$
2. $p^2 - 2p = 0$
3. $f^2 - 3f - 10 = 0$
4. $2y^2 + 4 = 9y$

Answers

1. $k = -6.3$ or $k = 0.3$
2. $p = 0$ or $p = 2$
3. $f = 5$ or $f = -2$
4. $y = 4$ or $y = 0.5$

Take a look at the working out for each answer **here**.

## Now What?

You may find it helpful to follow through and see how the quadratic formula is created. You can see the derivation in the **Maths is Fun** website at http://www.mathsisfun.com/algebra/quadratic-equation-derivation.html. It is not necessary to know this derivation, but it might assist your understanding. Another option for exploring the derivation of the quadratic formula is the video by **PatrickJMT** at http://patrickjmt.com/deriving-the-quadratic-formula/.

Now that you're familiar with the use of the quadratic equation, why not check your skills in some of the areas covered by the other algebra videos? **Factorisation of Algebraic Expressions** is particularly relevant to the alternative method of solving quadratic equations. Others such as **Algebraic Fractions** and **Simultaneous Equations** will also help develop your abilities in algebra.

## But When Am I Going To Use This?

Quadratic equations are very important as they are useful in understanding situations such as movement under constant acceleration (like gravity), a thrown ball if we ignore wind resistance, or pendulums or weights on springs. In fact, the quadratic equation is the base level solution for all stable systems, so it's one of the most widely studied systems in physics and engineering. Some stable systems that are often approximated using a quadratic equation include an atom's position in a molecule or solid, a thermostat attempting to maintain a constant temperature, or even a child on a swing.

## Other Links

As well as showing the derivation of the quadratic formula, **Maths is Fun** has a great applet where you can enter the formula for any quadratic equation. It then graphs the corresponding parabola for that quadratic and solves it for you. This is useful because it helps you visualise the solutions to the quadratic as positions on a parabola. However, keep in mind that you will still need to be able to solve the quadratic equation yourself.

- http://www.mathsisfun.com/quadratic-equation-solver.html

It also has worked solutions and sample questions.

- http://www.mathsisfun.com/algebra/quadratic-equation.html

**Mathportal** has another great applet to help develop your understanding of the quadratic equation and how to solve it. It allows you to input any quadratic equation, and shows a worked step-by-step solution using either the quadratic formula, or another method known as completing the square.

- http://www.mathportal.org/calculators/solving-equations/quadratic-equation-solver.php

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. It has a chapter dedicated to solving quadratic equations using the formula. This includes eight videos and a summarising test at the end.

- https://www.khanacademy.org/math/algebra/quadratics/quadratic-formula/v/using-the-quadratic-formula

**Patrick JMT** (Just Maths Tutorials) has a comprehensive set of video tutorials covering a large range of mathematical concepts. Here are two videos relevant to this topic: the first covers the use of the quadratic formula, and the second is the first in a series of videos showing example questions.

- http://patrickjmt.com/using-the-quadratic-formula/
- http://patrickjmt.com/solving-quadratic-equations-using-the-quadratic-formula-ex-1/

# Rates of Change and Differentiation

## Before You Watch

Before watching this video, make sure you've seen **Introduction to Calculus** first, as this video follows on from the introduction.

This topic also discusses some of the concepts that were introduced in **Linear Equations**, so consider watching that one as well, then come back.

## The Video Content

In the Introduction to Calculus, we considered a car moving, and we saw that its speed at an instant is given by:

$dx\ /\ dt$

where:

$dx$ means a tiny change in position

and:

$dt$ is a tiny change in time.

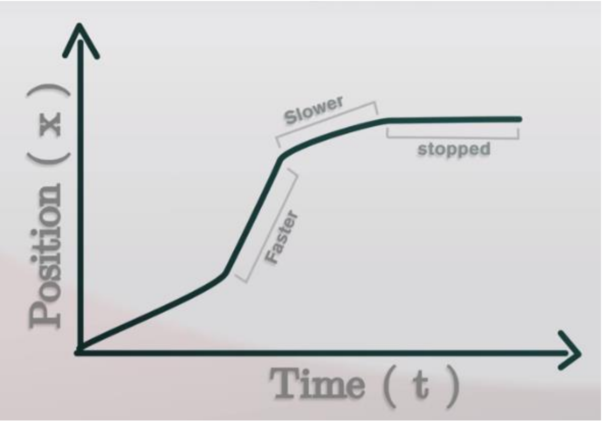
So, speed tells us how two things change together. It tells us how position changes as time changes.

This is what we call a *rate of change*.

Rates of change are all around us. Any time we compare two values and see how they change together, we are considering a rate of change. How much fuel does my car consume to drive 100 km? That's measuring a change in fuel against a change in position. How long until my beer is cold? That's measuring a change in temperature against a change in time. You get the idea.

Going back to the car problem, let's look at a graph of it.

The car's *position* is up the vertical axis, and the *time* is along the horizontal axis.




As the car moves across the graph, time goes by, and the position increases:

- if the car goes faster, the position changes more as time passes, so the line is steeper
- if the car slows down, the line is less steep
- if the car stops, then position doesn't change but time does, so the line is horizontal.

As you can see, the rate of change of the car's position versus time – also known as the car's speed – is related to the gradient.

Remember that *gradient* is *rise* over *run*?

In this example, the rise over any bit of the line is a change in position, and the run is a change in time.

So rise over run is:

change in position / change in time.

But we already know that change in position is $dx$ and change in time is $dt$.

Therefore gradient is the rate of change – they are the same thing!
In this example, the rate of change is speed.

### *Did you know?*

In this example the $y$ axis is position, and the $x$ axis is time, so the gradient is:

change in position / change in time = speed

Another way of thinking about this is to consider some example units of measurement. In this instance, position can be measured in metres, and time measured in seconds, so a suitable unit for the gradient is metres per second.

Or, position could be measured in kilometres, and time in hours, so the gradient would be measured in kilometres per hour.

At university you will come across many graphs and be asked to interpret the meaning of the gradient. The method, however, remains the same:

gradient = change in $y$ axis / change in $x$ axis

The units for this gradient can always be thought of as:

[units of $y$ axis] per [units of $x$ axis]

Let's say a car is travelling at 60 km/hr. The position of the car is given by:

$$x = 60t$$

where $t$ is the time in hours.

At one hour it’s 60 km away, at two hours it’s 120 km away and so on.

The speed is 60, so:

$$dx / dt = 60$$

The process of going from an equation for position, $x$, to an equation for speed, $dx / dt$, is called *differentiation*.

Therefore, to find the rate of change, which is the gradient, we need to differentiate.

It is important to remember that:

- *differentiation* is finding the *rate of change* between two values
- the *rate of change* is represented graphically by the *gradient*.

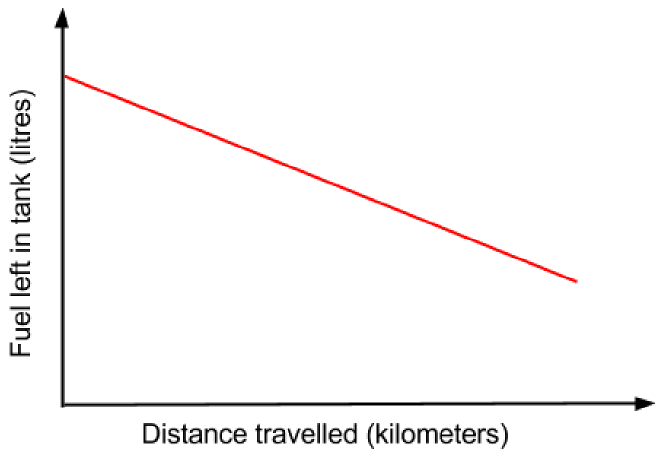
In this example, it might not seem necessary to differentiate. After all, we know the speed is 60 km/hr. But what happens if the car is accelerating?

In this situation, the speed, the gradient, is constantly changing. Let's think about calculating the gradient. If we pick any two points in time, we can find the average speed over this time, but that won't be the correct speed at either time.

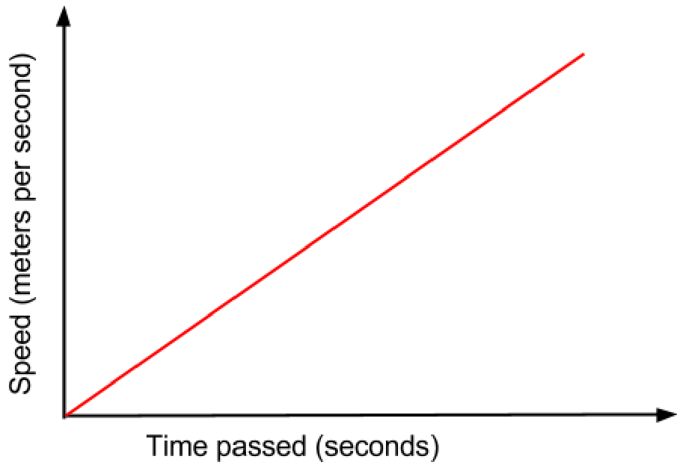
In this example, the formula of rise over run will fail. Therefore, to find the gradient at a point, we need to find the gradient of the tangent at that point. Mathematically, this is what differentiation is. We will look at this situation in the next topic.

## *Some Practice Questions*

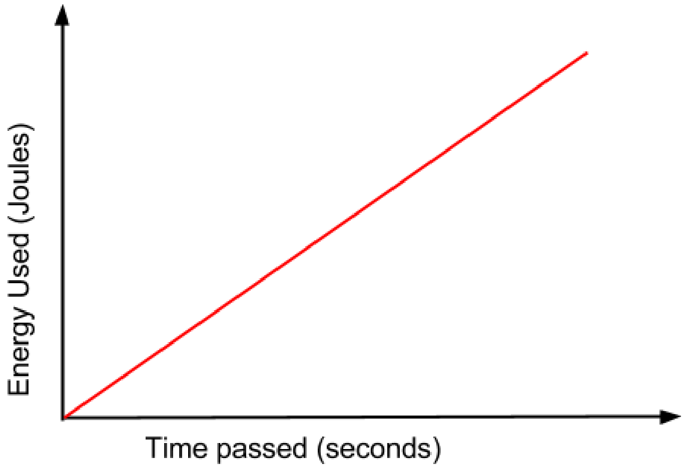
1. Read the graph below, and determine the correct *units* (not the value, the units) for the *slope (*or rate of change) of the graph.



2. Read the graph below, and determine the correct *units* (not the value, the units) for the *slope (*or rate of change) of the graph.




3. Read the graph below, and determine the correct *units* (not the value, the units) for the *slope (*or rate of change) of the graph.




Answers

1. Litres per kilometre (L/km). This rate of change is often used to measure the fuel economy of cars. Usually the distance is measured in hundreds of kilometres, so the unit would be litres per hundred kilometres.

2. Metres per second per second (m/s/s) *or* metres per second squared (m/$s^2$ *or* m$s^{-2}$). This is the standard scientific unit for measuring acceleration. When you think about it, we are measuring how speed changes over time, and that's acceleration.

3. Joules per second (J/s). This is also known as Watts, and is the standard unit of power, commonly used to measure, for example, the power of your car, heater or microwave. Power is therefore the rate of change between energy and time; it measures how much energy is used over time. Another way of putting it is that power measures how much time it takes to use energy.

## Now What?

By now you will be familiar the basics of calculus, the meaning of rates of change, and why we are interested in rates of change. You should also understand the concept of differentiation, which is the mathematical process of going from one formula that relates two variables (such as position and time) to another formula that gives the rate of change between those two variables (such as the rate of change between position and time, also known as speed).

The next step is to learn how to differentiate. In the next topic, **Differentiation of Polynomials**, you'll be shown how to differentiate the group of equations known as polynomials.

## But When Am I Going To Use This?

Calculus is the mathematical study of how things change relative to one another. For instance, velocity (or speed) is a change of position over a change in time, and acceleration is a change in velocity over a change in time – so any motion is studied using calculus. Other examples include the flow of water through pipes over time, or changing commodity prices against demand. Because change is everywhere, the potential applications for calculus are endless, particularly in engineering and science. Calculus is necessary knowledge for any degree related to engineering or science.

## Other Links

**Maths is Fun** has a great page that takes you through a simple problem which highlights the need for calculus to discuss changes happening around us. It then continues to explore the main two areas of calculus, differentiation and integration, and provides regular questions to test your understanding.

- https://www.mathsisfun.com/calculus/introduction.html

**IntMath** gives a bit of historical perspective to explain the sometimes confusing notation that is used in calculus, discussing how it is the mixed product of two mathematicians working independently. It also provides some excellent examples of applications of calculus that are in common use today, as well as helpful applets to understand both differential and integral calculus.

- http://www.intmath.com/calculus/calculus-intro.php

The **Khan Academy** has a comprehensive set of video tutorials covering a wide range of mathematical topics, as well as questions to test your knowledge. This content explains the historical development of calculus, and is also an excellent introduction to differential calculus and the concepts it is based around. From here you can further investigate differential calculus.

- https://www.khanacademy.org/math/differential-calculus/taking-derivatives/intro_differential_calc/v/newton-leibniz-and-usain-bolt

**Patrick JMT** (Just Maths Tutorials) has an extensive set of video tutorials covering a large range of mathematical concepts. This video introduces and explains the concept of a limit to help develop your understanding of this idea.

- https://patrickjmt.com/what-is-a-limit-basic-idea-of-limits/

# Simultaneous Equations

## Before You Watch

This topic follows directly on from Linear Equations. It covers how to take two linear equations and solve them together to get a single value for the two variables (which are usually $x$ and $y$, but could be any two letters). Thinking about this process graphically, we are finding the point where the two lines intercept. This is called solving simultaneous equations. Keep in mind that in Australia this topic is known as simultaneous equations, but in other countries it is often referred to as “solving a system of equations”.

If you haven't watched **Linear Equations** yet, take a look at that video first, then come back.

## The Video Content

Linear equations are used in a variety of applications. But what if you’re asked to solve two linear equations simultaneously? We’ll show you what this means, and how to do it. Let’s start with an example.

Say we’ve been asked to solve these equations simultaneously:

$$y = 3x + 2$$
$$2y + x = 26$$

So what does ‘solving simultaneously’ mean?

**Step 1 Understand the question**

A single linear equation has an infinite number of solutions, and these solutions can be represented by a line when we graph them. But if there are two linear equations, we can solve them together. This will give us a single solution, where the two lines cross!

That's what solving simultaneously means: to take two equations – each of which has an infinite number of solutions – and find a point which is a solution to both of them.

**Step 2 Develop a plan**

How do we do this? Let's go back to the two equations. Each of them has two letters. To solve this, we need to combine these two equations, to get an equation with only one letter in it. If an equation has only one letter, we can solve it to find a value for that letter.

**Step 3 Carry out the plan**

It helps to number the equations, so they are easier to keep track of:

$y = 3x + 6$ **(1)**

$2y + x = 26$ **(2)**

Equation **(1)** tells us that $y$ is equal to $3x + 6$. So, taking this value for $y$ from equation **(1)** and substituting it into equation **(2)**:

$2\,(3x + 6) + x = 26$

Now there are only $x$'s! Let's expand the brackets and simplify:

| | |
|---|---|
| $6x + 12 + x = 26$ | now bring the $x$'s together |
| $7x + 12 = 26$ | now take away 12 from both sides |
| $7x = 14$ | now divide both sides by 7 |
| $x = 2$ | now we know that $x = 2$ |

Next, to find the value of $y$, substitute $x = 2$ into one of the original equations, say equation **(1)**, which is $y = 3x + 6$:

$y = 3\,(2) + 6$

$y = 6 + 6$

$y = 12$

Now we know that $y = 12$. And that's our solution:

$x = 2$ and $y = 12$.

**Step 4 Reality Check**

To check that the answer is correct, substitute $x$ = 2 into both the original equations to see if $y$ = 12.

Yes, this works in both of them, so we know we have the correct solution.

## *Some Practice Questions*

Solve simultaneously:

1. 7 = 3$x$ + 2$y$, 4$x$ - $y$ = 3
2. 11 = 7$n$ - 4$r$, 5 + 12$n$ + $r$ = 11
3. 0.5$p$ = $r$ + 7, 2$p$ = 3$r$ + 7
4. 3 = $x$ / $y$, $y$ = $x$ + 4

Answers

1. $x$ = 13 / 11, $y$ = 19 / 11
2. $n$ = 81 / 52, $r$ = -207 / 104
3. $p$ = -28, $r$ = -21
4. $x$ = -6, $y$ = -2

Take a look at the working out for each answer **here**.

## Now What?

Here we've shown the simplest way of solving two linear equations simultaneously. In general, any two equations with the same two unknowns can be solved simultaneously. Bear in mind, however, that depending on the equations, that's sometimes not possible. Let's think about it graphically: each equation is a line, and if the two lines cross, you can solve them. If they don't cross, you won't be able to find a real solution.

Moving beyond two equations, what if there are three, or four equations? If you have more equations, you can find more unknowns.

Once you are comfortable with solving two equations simultaneously, consider giving three equations a go at:

- https://www.khanacademy.org/math/algebra2/systems_eq_ineq/fancier_systems_precalc/v/systems-of-three-variables

You should also consider looking at how to solve other equations, such as **quadratic equations** or equations with **exponentials**.

## But When Am I Going To Use This?

Linear equations are very commonly used in everyday life to model situations. So when two things are both modelled by a linear equation, we can solve them together (simultaneously) to find out important information.

A common example of this is in finance with the analysis of revenue and costs. The revenue for a company from a product (say $R$) is simply the price of the product (let's say \$30) *multiplied by* how many they sell ($n$), so:

$$R = 30n$$

This relationship can be represented by a straight line. Similarly, the cost of that product ($C$) is a fixed starting amount (the development cost, staff wages and so on) *plus* whatever the cost of making the product is *multiplied by* how many are sold.

Let's say there is a fixed monthly cost of \$1500 and the product costs \$10 to make. That means the costs per month are:

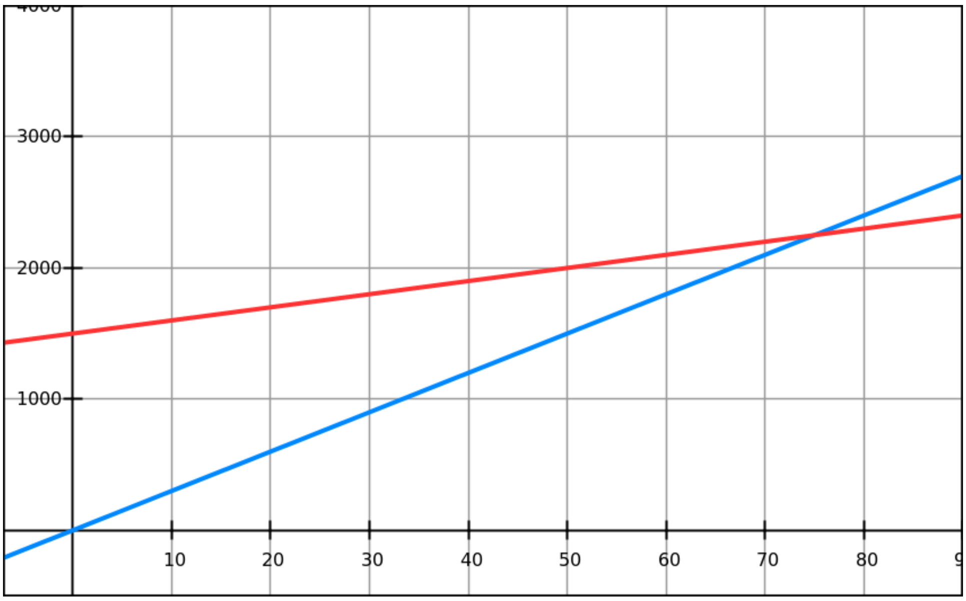
$$C = 1500 + 10n$$

This can also be represented as a straight line. So when revenue is equal to costs ($R = C$), the company is breaking even.

By solving these two equations simultaneously:

$$C = 30n$$
$$C = 1500 + 10n$$

we can determine how many products, $n$, must be sold per month for the company to break even. See if you can work it out, the answer is $n = 75$. A graph of the functions is given below.




This is just a single example. Some other examples can be found at:

- http://www.shelovesmath.com/algebra/intermediate-algebra/systems-of-linear-equations/

## Other Links

**She Loves Maths** has a wide range of pages covering many mathematical topics across all levels. The pages cover each topic using practical, relatable examples. This content deals with systems of linear equations.

- http://www.shelovesmath.com/algebra/intermediate-algebra/systems-of-linear-equations/

**Maths is Fun** provides a clear summary of solving equations and explains the different terminologies used. It also offers several questions to practise on.

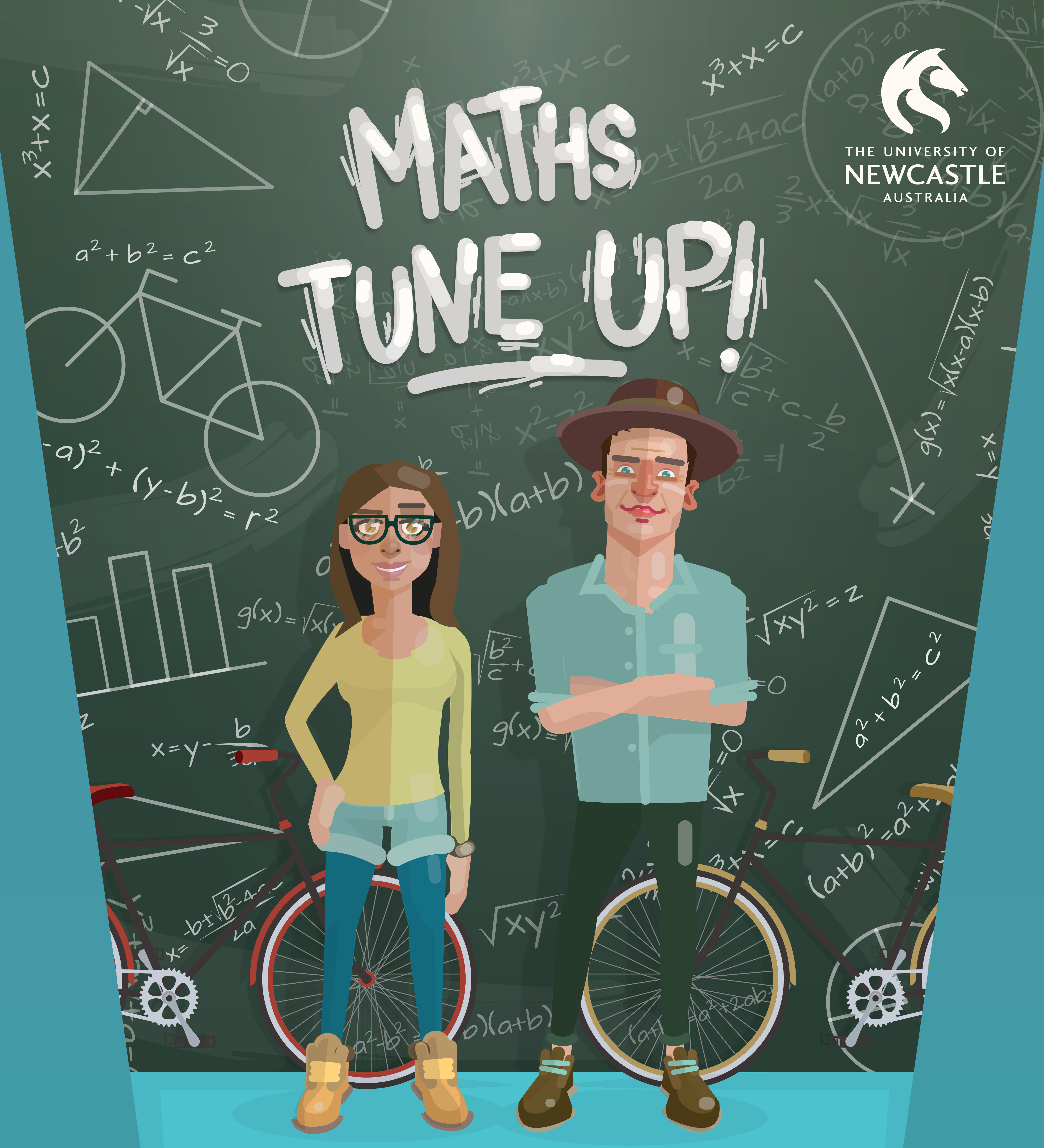
- http://www.mathsisfun.com/algebra/systems-linear-equations.html

The **Khan Academy** has a comprehensive set of video tutorials covering a large range of mathematical and other concepts, as well as questions to test your knowledge. This link is to a chapter dedicated to solving equations, with over a dozen videos and several quizzes.

- https://www.khanacademy.org/math/algebra2/systems_eq_ineq/systems_tutorial_precalc/v/trolls-tolls-and-systems-of-equations

**Patrick JMT** (Just Maths Tutorials) has many video tutorials covering a large range of mathematical concepts. The content below demonstrates three different techniques for solving a system of linear equations. Any of these methods can be used to solve any system of equations but, depending on the exact question, one strategy may be easier to use than the others.

- http://patrickjmt.com/row-reducing-a-linear-system-of-equations/
- http://patrickjmt.com/solving-a-linear-system-of-equations-using-elimination-by-addition/
- http://patrickjmt.com/solving-a-linear-system-of-equations-using-substitution/




To help give you a head start with your maths subjects, we've created a special series of videos.

Maths Tune Up is designed to get you up to speed quickly... because we all need a tune up every now and then!

**www.mathstuneup.com.au**

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# MATHS TUNE UP!

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www.mathstuneup.com.au

**MEDIA RELEASE**

**New maths videos and website designed to help students succeed at university**

A series of animated videos designed to help university students develop their maths skills is now available on the new Maths Tune Up! website created by the University of Newcastle.

The videos, which can be found at www.mathstuneup.com.au, cover a wide range of mathematical concepts that students need to understand in order to stay on top of their studies.

Each video provides a short reminder of a different mathematical concept, runs for just a few minutes, and includes easy-to-follow explanations and examples. As well as the videos, the Maths Tune Up! website features practice questions with solutions and working out, plus downloadable resource sheets that summarise each topic.

“One of the main advantages of Maths Tune Up! is that students don’t have to ask for help,” said Dr Elena Prieto-Rodriguez, lead academic on the project. “These resources are available 24/7 to anyone who wants to use them. Students can revisit some of the basic concepts in maths whenever and wherever they like and become independent learners.”

A sound knowledge of a range of mathematical concepts has often been identified as a major factor in enabling university students to gain access to and succeed in degrees involving science, technology, engineering and maths (STEM). However, many STEM students can do with some help to boost confidence with maths.

“For some students, maths can become a roadblock that stops them from continuing with their studies,” explained Dr Prieto-Rodriguez. “Even students who have previously done a lot of maths might need to review some of the basics. Maths Tune Up! is designed to give students easy access to the help they need to make progress in their studies.”

The Maths Tune Up! project was funded through the Commonwealth Government’s Higher Education Participation Programme (HEPP), which aims to improve access to higher education for people who would not otherwise attend university. This includes, for example, people from low SES backgrounds or who may be the first in their family to attend university. Once those students get to university, HEPP also helps them stay there and successfully complete their degree.

**For further information about Maths Tune Up! contact Dr Elena Prieto-Rodriguez on elena.prieto@newcastle.edu.au or phone 4921 7916**

# PROJECT EVALUATION - RESEARCH DESIGN

## 1 RESEARCH QUESTIONS

**From the HEPPP application:** This *project's objective* is to create, evaluate and disseminate a set of research informed digital media resources to help STEM university students from low SES backgrounds succeed in mathematics. These resources are centred on mathematical concepts known to be roadblocks for STEM students. These concepts are sometimes referred to as threshold concepts (Meyer, 2008; Meyer & Land, 2003).The project will draw on the experiences and expertise of teaching academics in Engineering, Education, Mathematics and Science, who recognise the challenges faced by students from low SES backgrounds and the possibilities for improving their experiences at tertiary level.

Aims:

1. To create resources that are research informed
2. Help STEM university students from low SES backgrounds succeed in mathematics
3. Improve their experiences at tertiary level

## 2 METHODOLOGY

The research will be divided into 3 phases

1. Creation of Resources,
2. Testing of Resources, and
3. Implementation of Resources.

A graphical representation of the Methodology as will be implemented for each Aim of the project can be found in Figure 1.

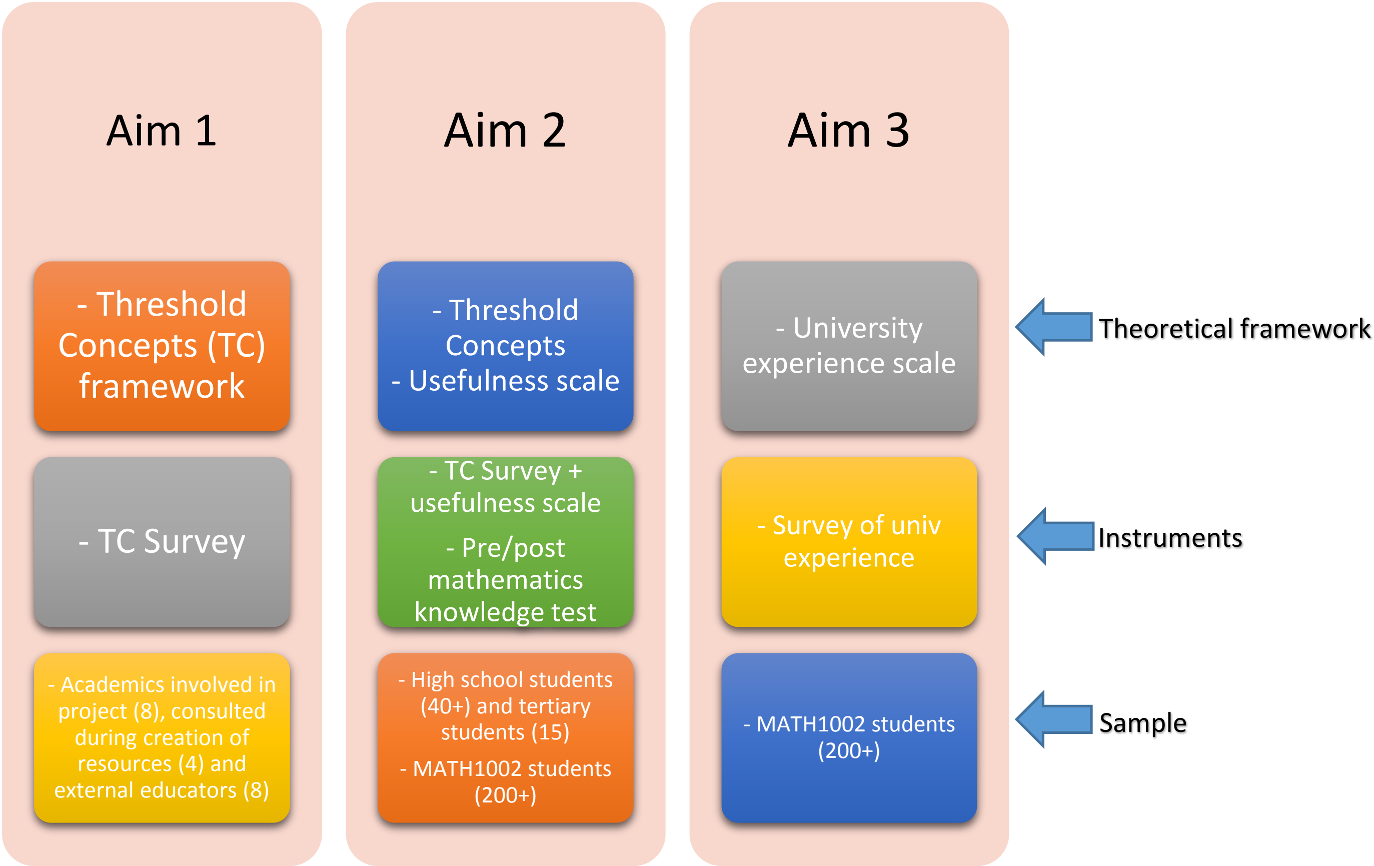


*Figure 1: Methodology*

## 2.1 Methods

**The first aim** of the project will be investigated in Phases 1 and 2 using the TC framework. To this end, all people involved in the creation of resources in Phase 1 will be surveyed as well as a group of educators and experts involved in testing in Phase 2. The instrument used will be Instrument 1 (see below). The survey will be online, anonymous, with some open ended questions and approximately 20 minutes in duration. It will be administered in September to academics involved in Phase 1 and Educators in Phase 2.

**The second aim** of the project will be investigated in Phases 2 and 3 using both the TC framework and a usefulness scale.

Students involved in the testing of the resources in Phase 2 will be tested on mathematical knowledge before and after watching the videos (students will respond to Instrument 2.1 before watching the videos, and after watching them, they will respond to Instrument 2.2.). The test and associated short survey will be paper-based and will contain expanded response mathematical items (short, but not multiple choice) and Likert-type items. It will take approximately 30 minutes to complete each of the tests.

In the case of students in Phase 2, the testing phase, the time between the tests will be three weeks. Also, focus groups will also be conducted to test the appropriateness of the videos. In this case the Threshold Concepts framework will be used.

In the case of students in Phase 3, the implementation phase, Instrument 2.1 will be delivered before Week 3 and Instrument 2.2 after the mid-semester break.

**The third aim** of the project will be investigated in Phase 3 using surveys and interviews about their university experience. Students participating in Phase 3 of the project will be surveyed about their university experience and the role of mathematics in it. Given the larger sample size for Phase 3 a survey might be sufficient and then target a few students for interviews based on their survey responses – some who liked the videos and accompanying online resources, some who were indifferent and some who didn't find them useful.

## 2.2 Sample by phase

### 2.2.1 Phase 1:

- Academics involved in the project (8)
- Academics and teachers consulted during creation of resources (6)

### 2.2.2 Phase 2:

- Educators
    - People involved in education in the Hunter region (4+).
    - IMSITE education academics (4+).
- Students
    - Students from local high school (40+)
    - Students in university Enabling program, EPMTH309, EPMATH125 courses (15+)
    - Students in first year university Engineering program, Bill McBride's course (40+)

### 2.2.3 Phase 3:

- Sample of science and engineering students in MATH1002 (200+)

## 2.3 Instruments:

### 2.3.1 Instrument 1:

Substantive survey using Threshold Concepts framework.

Sample questions:

| | |
|---|---|
| Which of the videos do you consider present knowledge that is **transformative**? | TRANSFORMATIVE KNOWLEDGE: The video can potentially involve a conceptual shift in the way students think about the mathematical topic |
| Which of the videos do you consider present knowledge that is **irreversible**? | IRREVERSIBLE KNOWLEDGE: The video's concept, once understood, will not be forgotten |
| Which of the videos do you consider present knowledge that is **integrative**? | INTEGRATIVE KNOWLEDGE: The video exposes the interrelatedness of the concept to other concepts in mathematics or STEM |
| Which of the videos do you consider present knowledge that is **bounded**? | BOUNDED KNOWLEDGE: The video delineates a conceptual space. Boundedness may be associated with a discipline's special language |
| Which of the videos do you consider present knowledge that is **troublesome**? | TROUBLESOME KNOWLEDGE: The concepts the video is trying to explain may appear to students as counter-intuitive or 'hard' |

| | |
|---|---|
| Did you find video/resource #X useful? | |
| Would you recommend video/resource #X to other people? | |
| Would you use video/resource #X in your teaching practice? | |
| Positive characteristics of the videos | |
| Negative characteristics of the videos | |
| Other videos | |
| Areas of improvement | |

### 2.3.2 Instrument 2:

#### *2.3.2.1 Instrument 2.1*

Short mathematics knowledge test + demographics.

Students are advised how to create unique anonymous and easily repeatable identifier in order to match pre- and post- data.

#### *2.3.2.2 Instrument 2.2*

Short mathematics knowledge test (2) + additional questions about how much they have like videos or found them useful.

Students are advised how to use their unique anonymous and easily repeatable identifier in order to match pre- and post- data.

| | |
|---|---|
| Did you find the video useful? | |
| Would you recommend it to other people? | |
| Areas of improvement | |
| Positives | |
| Negatives | |

### 2.3.3 Instrument 3:

Questions about mathematics experience at tertiary level.

Students are advised how to use their unique anonymous and easily repeatable identifier in order to match pre- and post- data.

| | |
|---|---|
| Are you enjoying university life? | |
| Are you enjoying learning mathematics? | |
| | |
| | |

# 3 ANALYSIS

- Consider differences (before and after video/resources intervention)
- Consider relationships between mathematical knowledge/usefulness of videos and resources/enjoyment of university life

## References

Meyer, J. (2008). *Threshold concepts within the disciplines*: Sense publishers.

Meyer, J., & Land, R. (2003). *Threshold concepts and troublesome knowledge: linkages to ways of thinking and practising within the disciplines*: University of Edinburgh Edinburgh.

Meyer, J., & Land, R. (2003). *Threshold concepts and troublesome knowledge: linkages to ways of thinking and practising within the disciplines*: University of Edinburgh Edinburgh.

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## Maths Tune Up Feedback

### Introduction and welcome

**Welcome to the Maths Tune Up feedback page!**

**We are looking for ways to improve our design by having your opinion and we want some honest feedback. We are not testing you, or your knowledge, but the product.**

**While not all features are enabled we would like your feedback on the existing layout and content: this includes two 3-minute videos and associated resources.**

**You may exit at any time by closing your browser. Click next to continue.**

1. Please enter your school name.

2. What year are you in at school?

- ◯ Year 11
- ◯ Year 12
- ◯ N/A

3. What is your gender?

- ◯ Male
- ◯ Female
- ◯ Other

## The website

**Please indicate your level of agreement with each of the following items.**

4. The layout of the website was user-friendly.

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

5. The advice within ‘Before you watch’, ‘Now what’, ‘But when am I going to use this’ was helpful.

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

6. Identify what you liked best about the website.

7. Identify anything you feel could be improved about the website

Maths Tune Up Feedback

## The videos

**Please indicate your level of agreement with each of the following items.**

8. I feel the videos would assist those having difficulty with the topic area.

Extremely agree — Neutral — Extremely disagree

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

9. The characters made the videos more interesting than without them.

Extremely agree — Neutral — Extremely disagree

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

10. The videos were appealing.

Extremely agree — Neutral — Extremely disagree

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

11. I would recommend the videos to other people

Extremely agree — Neutral — Extremely disagree

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

12. Identify what you liked best about the videos.

13. Identify anything you feel could be improved about the videos.

Maths Tune Up Feedback

## The Practice Questions

**Please indicate your level of agreement with each of the following items.**

14. The practice questions were very useful.

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

15. The interactive nature and provided solutions made me want to see similar resources for other topics.

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

16. Identify what you liked best about the Practice Questions.

17. Identify anything you feel could be improved about the Practice Questions.

Maths Tune Up Feedback

## Comparison with other supporting resources

18. Have you accessed other online video/resource assistance for maths like these before?

- ◯ Yes
- ◯ No

**Maths Tune Up Feedback**

19. I found those other videos/resources useful

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

20. Please name any/all of those other video/resources that you can recall.

21. You preferred the format and content of today's viewed videos/resources.

| Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

## Branding

**Please rate each of the following items with regard to your preference for the name of the resources.**

22. Please rate each of the following names for the resources.

| | Extremely agree | | | | | Neutral | | | | | Extremely disagree |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ‘Yeah, Maths!’ would be a good branding for these resources | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ |
| ‘UniTunes’ would be a good branding for these resources | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ |
| ‘Newy Maths’ would be a good branding for these resources | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ |
| ‘Tune In: Maths’ would be a good branding for these resources | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ |
| ‘Maths Tune Up’ would be a good branding for these resources | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ | ◯ |

## End of survey

**Thank you for helping!**


# PRACTICE GUIDE: IDEAS FOR USING MATHS TUNE UP!

## WHAT IS MATHS TUNE UP!

A series of animated videos designed to help university students develop their maths skills is now available on the new Maths Tune Up! website created by the University of Newcastle. The videos cover a wide range of mathematical concepts that students need to understand in order to stay on top of their studies.

Each video provides a short reminder of a different mathematical concept, runs for just a few minutes, and includes easy-to-follow explanations and examples.

### What resources are on the website?

As well as the videos, the Maths Tune Up! website features practice questions with solutions and working out, plus downloadable resource sheets that summarise each topic. Maths Tune Up! is designed to give students easy access to the help they need to make progress in their studies.

**Q: Where can I find the website?**
A: The website can be found at www.mathstuneup.com.au

**Q: What topics are covered on the website?**
A: The videos are categorised into three mathematical strands required in engineering fields as well as 'numeracy development' and 'thinking mathematically' strands.

*Results from our online survey indicate that 97% of respondents felt the videos would assist those having difficulty with the topic area.*

| | |
|---|---|
| Problem-Solving | •Introduction to Problem-Solving |
| Algebra | •Introduction to Algebra<br>•Linear Equations<br>•Simultaneous Equations<br>•Indices Laws<br>•Negative and Fractional Indices<br>•Factorisation of Alegraic Equations<br>•Quadratic Equations<br>•Algebraic Fractions<br>•Exponentials and Logarithms |
| Trigonometry | •Introduction to Trigonometry<br>•Components of Vectors |
| Calculus | •Introduction to Calculus<br>•Rates of Change and Differentiation<br>•Differentiation of Polynomials<br>•Integration |
| Probablity | •Introduction to Probability<br>•Introduction to Statistics |
| Proofs | •Introduction to Proofs |

## HOW CAN I USE THE RESOURCE WITH MY STUDENTS

### In the classroom

The resource can be used in the following ways:

1. Embedded into course material and learning activities: Learners could utilise the videos as a self-diagnostic in order to identify areas for improvement and complete the interactive Practice Questions to gauge improvement. In this way, the website functions as a formative assessment tool. In addition, the website can be accessed via smartboards and projected for instructor demonstration or collaborative learning.

2. Just-in-time learning tools: The website can be used as a learning tool to support mathematical improvement within a cohort, if knowledge gaps have been identified. If gaps are identified as part of the formative and summative assessment within a course, the instructor can direct students to the site for self-paced numerical improvement.

3. Specialist study skills workshops or academic preparation courses: The website can be included as a key learning tool in the academic numeracy component of study skill or academic preparation courses. Instructors can set learners the task of viewing the videos in order to promote self-diagnosis of

mathematical gaps and aid reflective self-assessment of improvement to successfully meet the numeracy requirements of their course.

### Out of the classroom

The website was developed to support mobile learning (m-learning). This involves students using the site for independent learning at their own pace, at anytime and anywhere. The website can be promoted as an m-learning tool as part of resources in a course, degree program or learning development unit.

## ACKNOWLEDGEMENTS

The Maths Tune Up! project was funded through the Commonwealth Government's Higher Education Participation Programme (HEPP), which aims to improve access to higher education for people who would not otherwise attend university. This includes, for example, people from low SES backgrounds or who may be the first in their family to attend university. Once those students get to university, HEPP also helps them stay there and successfully complete their degree.

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*For further information about Maths Tune Up! contact Dr Elena Prieto-Rodriguez on elena.prieto@newcastle.edu.au or phone 4921 7916*

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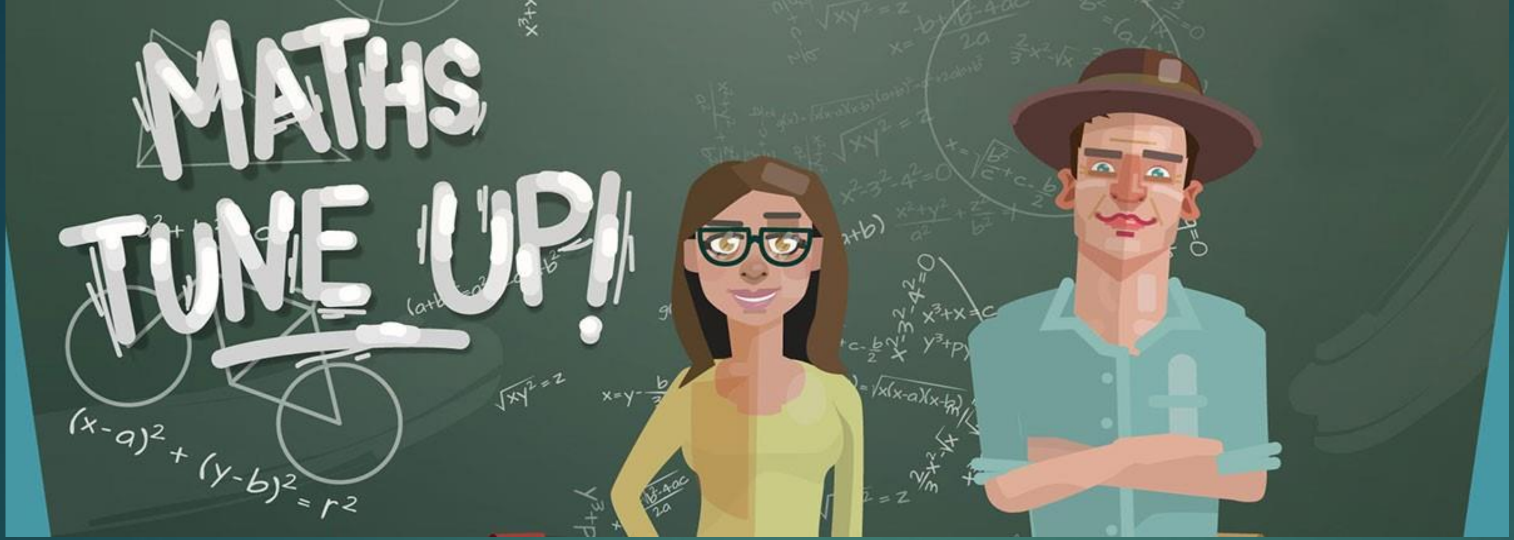



THE UNIVERSITY OF NEWCASTLE AUSTRALIA

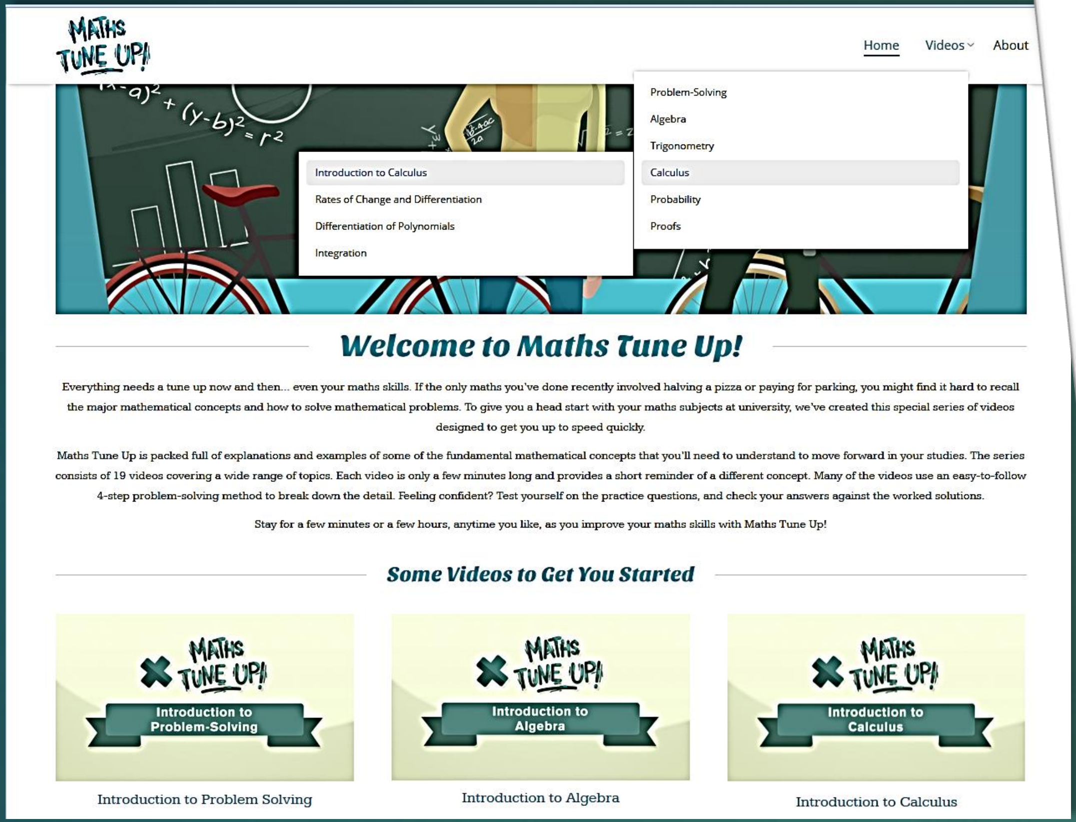
# What is Maths Tune Up?

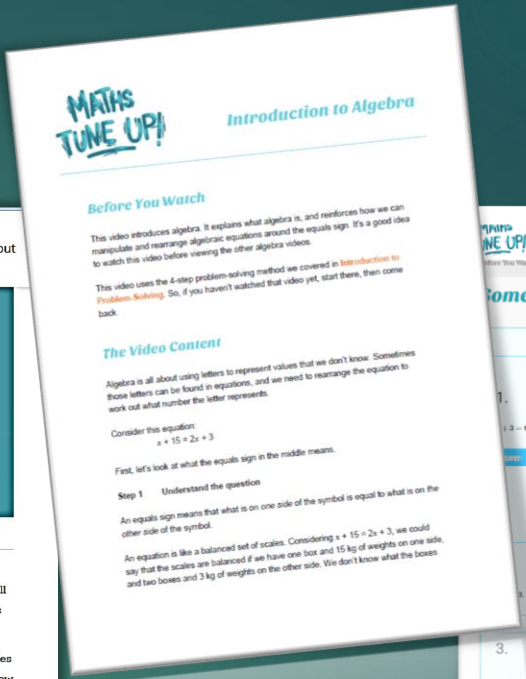
Maths Tune Up! is a new website that has a series of animated 3-minute videos designed to help university students develop their maths skills.

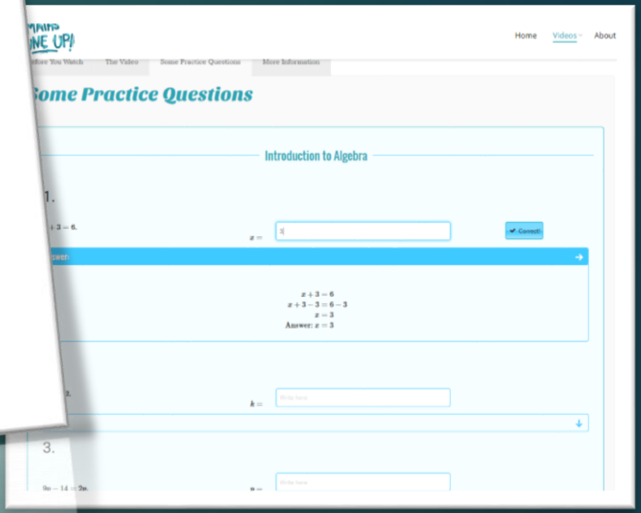
The videos cover a wide range of essential mathematical concepts that first year students need in order to progress with their studies.

Each video is only a few minutes long and provides a short reminder of a different concept.

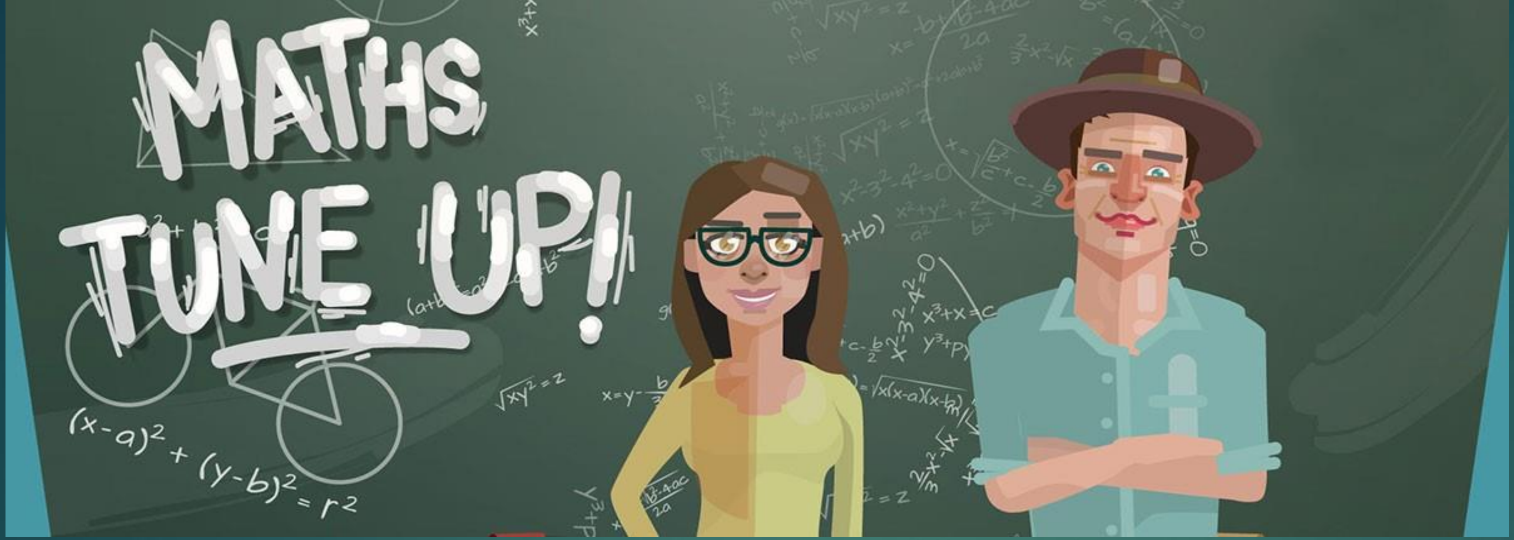
# What else does it have?




The website also features practice questions with full working out, plus downloadable resource sheets that summarise each topic.




## Where can I find it?

You can find the Maths Tune Up! website plus a short video introduction to the resource here:

**www.mathstuneup.com.au**

Maths Tune Up! is an initiative of the University of Newcastle and funded through the Commonwealth Government's Higher Education Participation Program (HEPP).

## PRESENTATION GUIDE FOR LECTURES OR TUTORIALS

**New maths videos and website designed to help students succeed at university**

A series of animated videos designed to help university students develop their maths skills is now available on the new Maths Tune Up! website created by the University of Newcastle. The videos, which can be found at www.mathstuneup.com.au, cover a wide range of mathematical concepts that students need to understand in order to stay on top of their studies.

Each video provides a short reminder of a different mathematical concept, runs for just a few minutes, and includes easy-to-follow explanations and examples. As well as the videos, the Maths Tune Up! website features practice questions with solutions and working out, plus downloadable resource sheets that summarise each topic. Maths Tune Up! is designed to give students easy access to the help they need to make progress in their studies.

The Maths Tune Up! project was funded through the Commonwealth Government's Higher Education Participation Programme (HEPP), which aims to improve access to higher education for people who would not otherwise attend university. This includes, for example, people from low SES backgrounds or who may be the first in their family to attend university. Once those students get to university, HEPP also helps them stay there and successfully complete their degree.

**Suggested use of the site and introduction**

1. GO TO SITE:

www.mathstuneup.com.au

2. SUGGESTED NARRATIVE:

I'd like to tell you about a new website that contains a series of animated videos designed to help develop your maths skills. The site is Maths Tune Up! and has been created by the University of Newcastle. The videos cover a wide range of mathematical concepts that you need to understand in order to stay on top of your studies.

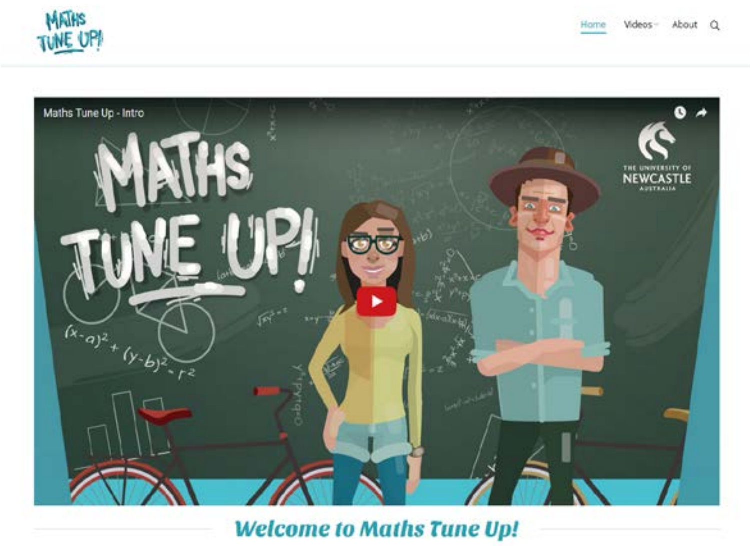
Each video provides a short reminder of a different mathematical concept, runs for just a few minutes, and includes easy-to-follow explanations and examples. As well as the videos, the website features practice questions with solutions and working out, plus downloadable resource sheets that summarise each topic.

One of the main advantages of Maths Tune Up! is that you don't even have to ask for help. The resources are available 24/7 to anyone who wants to use them. You can revisit some of the basic concepts in maths whenever and wherever you like.

Let's take a look.

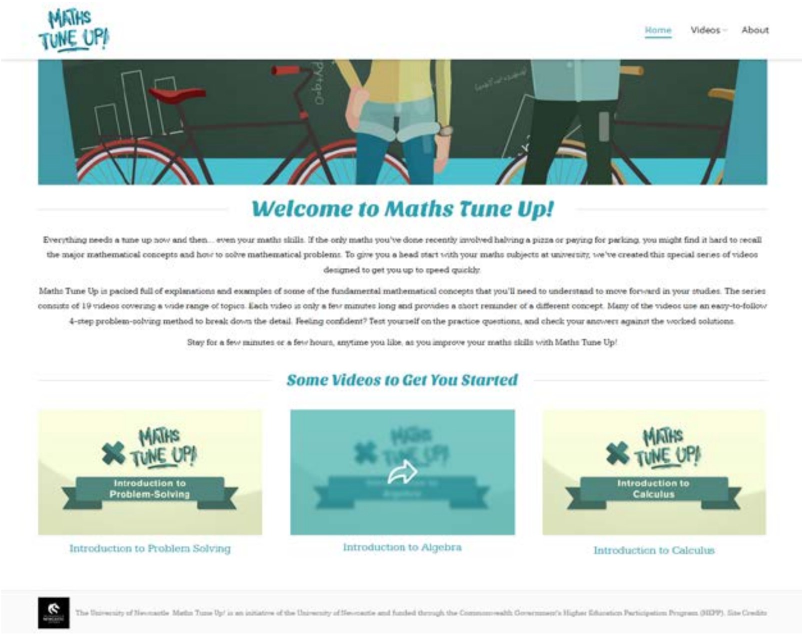
3. PLAY TEASER VIDEO:

Running time 1:00 minutes




4. PLAY INTRO TO ALGEBRA VIDEO

Running time 2:54




5. EXPLORE other tabs on the site after viewing the video:

- Resource Sheet (download)
- Some Practice Questions
- More Information

**Further information**

The results from an online evaluation completed by 61 students demonstrated that 97% of respondents felt the videos would assist those having difficulty with the topic area and considered the layout of the website was user friendly.

**For further information about Maths Tune Up! contact Dr Elena Prieto-Rodriguez on elena.prieto@newcastle.edu.au or phone 4921 7916**